On the Problem of Controlling Shiftable Prosumer Devices with Price Signals

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Abstract—Price-based control approaches can be derived from the Lagrangian relaxation of the unit commitment problem and provide a scalable framework for solving practical sized optimization problems. The framework postulates an iterative gradient based-pricing mechanism, in which the price signal is used to negotiate the energy exchange between individual market participants. Within this framework shiftable prosumer devices induce non-convex, non-continuous utility functions, which can cause divergence of the gradient based pricing mechanism resulting in market failures. This paper proposes a new simple but efficient method to deal with this problem by exploiting the concept of randomized price offsets.

I. INTRODUCTION

Market-based control frameworks have gained a lot of interest during the last decade. In the area of automatic control the framework provides a scalable distributed approach in order to optimize the unit commitment of distributed generation units and demand response resources. Due to the indirect control mechanism and the inherent support for local decision making, market-based control frameworks can be applied across legal and organizational boundaries, which makes them especially suitable to be implemented as a system wide demand and supply management in liberalized energy market systems. In the context of smart markets this is crucial to cope with the increased short-term balancing deviations induced by forecast errors and higher variability of wind- and solar energy generation.

In this paper we deal with the general equilibrium problem for multi-commodity markets, which can be directly derived from the Lagrange Relaxation of the primal unit commitment problem by interpreting the Lagrange factors as negative market prices [1]. Based on this mathematical foundation a pricing mechanism known as the “Tantonnement Process” can be defined in order to search for the market equilibrium [2]. Thereby the price information consists of a price vector with a finite number of elements, in which the k-th element denotes the energy price in the corresponding k-th time interval (e.g. 15 min) defined over a finite optimization horizon (e.g. 24 h).

At the start of each period an iterative negotiation process is triggered, in which each market participant negotiates its predicted energy surplus/ deficit for current and future time periods. The iterative pricing process can be summarized as follows: in each iteration the so called market operator adjusts the price signal and distributes it to all market participants, which respond with their preferred energy schedule derived from their individual utility functions and the received price information. Based on the aggregated prosumer schedules and their individual price sensitivities (partial derivatives of the utility functions) the market operator updates the price information. This process continues until the market is cleared and a market equilibrium is established.

While this framework is widely accepted and discussed in public literature [3], there still exists a lot of open research questions regarding the solution quality and stability of the “Tantonnement process” when the framework is applied in a practical context like power systems with realistic consumer/producer behavior. The solution quality and stability e.g. greatly depends on the actual characteristics of the utility functions. For convex continuous utility functions it can be shown that the optimal solution to the dual problem corresponds with the optimal solution of the primal unit commitment problem. On the other side discontinuance can lead to instability of the Tantonnement process and therefore either to no – or suboptimal solutions [4], [5]. In the energy domain this is especially challenging because it is well known that many producers or generators have non-convex discontinuous utility functions. P. Papadaskalopoulos and G. Strbac, recently showed that this property also holds for many flexible loads and storage devices [6], [7].

In section II we will investigate and classify the characteristics of prosumer utility functions, which are typical for future power systems, e.g. renewable energy producers and shiftable consumers. We analyze the impact of these utility functions on the stability of the market mechanism by constructing an unstable reference scenario. The scenario consists of a variable size of market participants, which indicates that the instability still holds even if the market is sufficiently large.

In section IV we will present a simple, yet powerful variation of the market mechanism, which drastically improves the convergence behavior. The advantage over other methods is that our method is solely based on a slight variation of the price signal and doesn’t add extra communication or processing overhead. Also the prosumer utility functions resp. the local device control logic can remain untouched, which retains simplicity and transparency. An interesting property of the proposed method is that the solution quality and stability increases with the market size.

II. MARKET BASED OPTIMIZATION FRAMEWORK

In this section we briefly describe the market-based optimization approach for the unit commitment problem, present the standard iterative solution technic and discuss limitations concerning different prosumer classes.
A. Problem formulation

The Unit Commitment Problem can be defined as:

**Definition 1:** Given a set $I = \{i \in \mathbb{N} | 1 \leq i \leq n\}$ of controllable generation units and demand response resources with individual cost functions $\text{cost}_i(\cdot): \mathbb{R}^k \rightarrow \mathbb{R}$ as well as operational constraints $\mathcal{N}_i \subseteq \mathbb{R}^k$. The unit commitment problem (UCP) for $k$ time periods comprises the task of computing $n = |I|$ energy schedules $x_i \in \mathbb{R}^k$, such that:

$$\arg\min_{x_i \in \mathcal{N}_i} \sum_{i \in I} \text{cost}_i(x_i) \quad \text{s.t.} \quad \sum_{i \in I} x_i[j] = 0; \quad \forall j: 1 \leq j \leq k \quad (1)$$

Equation (2) represents the equilibrium constraints and $x_i[j]$ the energy allocation of the $i$-th unit in period $j$.

B. Market based solution approach

A common way to solve the UCP for practical sized problems is to decompose it into its dual representation using Lagrange relaxation. Therefore the equilibrium constraints (2) have to be integrated into the minimization function. By introducing a Lagrange factor $\lambda[j], \lambda \in \mathbb{R}^k$ for each equilibrium constraint the Lagrangian Dual $L_D$ is given by:

$$\max_{\lambda \in \mathbb{R}^k} \left\{ L(\lambda) := \arg\min_{x_i \in \mathcal{N}_i} \left\{ \sum_{i \in I} \text{cost}_i(x_i) - \sum_{i \in I} \lambda \cdot x_i \right\} \right\} \quad (3)$$

Obviously a solution $\Lambda^*$ for the dual form has the property $\partial L/\partial \lambda = 0$, which is a direct consequence of the maximization operator. Furthermore since $\partial L/\partial \lambda = 0$ the equilibrium constraints $\sum_{i \in I} x_i[j] = 0$ are also satisfied. The opposite direction is true if the cost functions are continuous and convex and thus imply a unique optimum. Another important property of (3) is the separability of the inner optimization problem. Especially given a specific $\lambda$, $L(\lambda)$ can be separated in $n = |I|$ independent minimization problems solvable in parallel.

In the context of economic calculations the dual form (3) is often formulated as the general equilibrium problem by interpreting the negative Lagrange factors as market prices $p := -\lambda$.

**Definition 2:** Given a set $I = \{i \in \mathbb{N} | 1 \leq i \leq n\}$ of controllable generation units and demand response resources with individual cost functions $\text{cost}_i(\cdot): \mathbb{R}^k \rightarrow \mathbb{R}$ as well as operational constraints $\mathcal{N}_i \subseteq \mathbb{R}^k$. The general equilibrium problem (GEP) comprises the task of finding a price vector $p = (p_1, \dotsc, p_k)$, such that:

$$\sum_{i \in I} u_i(p)[j] = 0 \quad \forall j: 1 \leq j \leq k \quad (4)$$

$$u_i(p) = \arg\min_{x_i \in \mathcal{N}_i} \text{cost}_i(x_i) + p \cdot x_i \quad \forall i : i \in I \quad (5)$$

Do note that equation (4) is directly derived from $\partial L/\partial \lambda = 0$. The functions $u_i(\cdot)$ are called utility functions of the $i$-th market participant.

Although there is no real difference between the dual form of the UCP and the GEP, the formulation given in definition 2 has the benefit of assigning a financial value to the traded resources, which can be used to organize and justify the trading process across multiple market participants in a competitive environment.

C. The Tatonnement process

From (4) we can observe that computing a solution for the GEP is equal to finding the root of a $k$ dimensional function. An efficient method of doing this is given by the Newton Raphson method, which leads to the following algorithm:

**Require:** $p_{\text{init}} \in \mathbb{R}^k, j_{\text{max}} \in \mathbb{N}, \epsilon_{\text{threshold}} \in \mathbb{R}$

**Ensure:** $j_{\text{max}} > 0 \land \epsilon_{\text{threshold}} > 0$

1: $j \leftarrow 0$
2: $p \leftarrow p_{\text{init}}$
3: $x_\Delta \leftarrow (\infty, \dotsc, \infty)^T$
4: **while** $x_\Delta \geq \epsilon_{\text{threshold}} \land j \leq j_{\text{max}}$ **do**
5: $\text{broadcast}(p, i) \forall i \in I$ \quad // compute $x_i, J_i$ in parallel
6: $x_\Delta \leftarrow \sum_{i \in I} \text{recv}(x_i, i)$
7: $J \leftarrow \sum_{i \in I} \text{recv}(J_i, i)$
8: $p \leftarrow p + \alpha \cdot J^{-1} \cdot x_\Delta$
9: $j \leftarrow j + 1$
10: **end while**

Here the methods broadcast($\cdot, i$) and recv($\cdot, i$) broadcasts/receives the information given by the first argument to/from the $i$-th market participant. The Jacobian Matrix $J$ contains the partial derivatives of the utility functions. For practical implementations especially if the utility functions are not continuous differentiable numerical differentiation can be used. Also various step size $\alpha$ heuristics for line (8) can be applied to further improve the convergence behavior [8].

D. Instability of the Tatonnement process in case of non-convex non-continuous utility functions

In this section we summarize the classes of utility functions that are hard to solve by the algorithm given in section II-C. The classification results are partly derived from reviewing the related literature and a dedicated case study [9], [10].

The first class $C_1$ consists of utility functions of storage devices and shiftable loads. To motivate this classification lets consider the bidding behavior of an electric vehicle. At the beginning of the charging process $t_{\text{start}}$ an electric vehicle typically wants to charge a fixed amount of energy $e_{\text{const}}$ before it leaves the charging spot in period $t_{\text{end}}$. While it is connected to the grid it will always charge at time periods with the lowest market prices. This behavior can be modeled by the following utility function $u(p)$:

$$u(p) = \arg\min_{x \in [0, p_{\text{max}}]^k} p \cdot x \quad \text{s.t.} \quad \sum_{j=t_{\text{start}}}^{t_{\text{end}}} x[j] = e_{\text{const}} \wedge \quad (6)$$

$$x[j] = 0 \quad \forall j : j < t_{\text{start}} \lor j > t_{\text{end}} \quad (7)$$

Storage devices exhibit a similar behavior by charging at time periods with low market prices and discharging at time periods with high market prices if the price margin between those time periods is higher than the storage operation costs. Similar classes have been identified in [6], [7].

The second class $C_2$ represents the class of utility functions of binary commitment devices. This class includes all devices...
with fixed marginal costs. An example for this class are μ-CHP units (see [11]). Another example are wind- or PV units with neglectable marginal costs. Both device types result in a bidding behavior, in which they will sell their maximal available power \( P_{\text{max}} \) as long as the market price is above the marginal costs and sell nothing when it doesn’t.

\[
u(p)[j] = \begin{cases} -P_{\text{max}} : & p[j] > \text{threshold} \\ 0 : & p[j] \leq \text{threshold} \end{cases} \tag{8}
\]

E. Market failure

We define a market failure for a market \( m \) if the algorithm proposed in section II-C is unable to find a market clearing price in a finite amount of steps. For practical applications especially in a smart market/ grid environment with millions of small participants the iteration count has to be kept low because the number of iterations is proportional to the number of messages that need to be exchanged between the market operator and all participants at the beginning of each period. For markets with a share of class \( C_1 \) resp. \( C_2 \) utility functions the algorithm tends to oscillate around the market clearing price while at the same time the market outcome remains at an unacceptable distance away from the market equilibrium (see figure 1).

To give an example lets consider a market with \( n_{EV} \) electric vehicles as shiftable loads and a reference generation unit with a quadratic cost term \( a \cdot x[j]^2 \), where \( a \) models the generators fuel costs. Thus the generator’s utility function according to (5) is given by:

\[
u(p)[j] = \text{argmin}_{x[j] \in \mathbb{R}} a \cdot (x[j])^2 + p[j] \cdot x[j] \tag{9}\]

For simplification let’s further assume the optimization horizon is limited to only two time periods \( t_1, t_2 \) and all electric vehicles are available for both time periods but only one period to fully charge e.g. \( c_{\text{const}} = P_{\text{max}}, t_{\text{start}} = t_1, t_{\text{end}} = t_2 \). The first thing to notice is that whenever \( p[t_1] = p[t_2] \) the charging behavior of the EVs is undefined in the sense that there exist two solutions (either charge at period \( t_1 \) or \( t_2 \) with the same charging costs. In order to maintain a well defined market equilibrium we assume that whenever the market price is equal in both time periods, the electric vehicles charge equally distributed either at period \( t_1 \) or at period \( t_2 \). The bidding behavior of the generators according to (9) is \( u_i(p) = (p[t_1], p[t_2])^T \) thus the market clearing price is \( p^* = (a n_{EV} P_{\text{max}}, a n_{EV} P_{\text{max}})^T \), for which the generators produce exactly \( x^* = \frac{n_{EV} P_{\text{max}}}{a} \) energy per period.

Given this market setup steps (5)...(9) of the proposed algorithm lead to the following trajectory of the market price \( p^j = \Delta p^j + p^* \):

\[
p^{j+1} := \begin{cases} s_1 \Delta p^{j}[t_1] + p^{*}[t_1] \\ s_2 \Delta p^{j}[t_2] + p^{*}[t_2] \end{cases} + \alpha^j 2a \begin{cases} s_2(x^* + \frac{\Delta p^{j}[t_1]}{2a}) \\ s_1(x^* + \frac{\Delta p^{j}[t_2]}{2a}) \end{cases} \tag{10}\]

with \( s_1 := \text{sgn}(\Delta p^j[t_1] - \Delta p^j[t_2]) \) and \( s_2 := -s_1 \).

Here \( \alpha^j \) is the step size parameter in the \( j \)-th update step. The only way to reach the market equilibrium price is an iteration with the step size \( \alpha^j = 1 \), which leads to \( |\Delta p^j[t_1]| = |\Delta p^j[t_2]| \) and a following iteration with a step size of:

\[
\alpha^k = \frac{\Delta p^0}{2ax^* + \Delta p^0}; \quad j < k \tag{11}\]

The latter choice of \( \alpha \) is difficult to be made by any stepsize heuristic because the market operator doesn’t know about the exact structure of the aggregated utility functions. Especially if the optimization framework is applied in a liberalized market context the information exchange is limited to the price signal and the corresponding partial derivates (here \( \frac{1}{2\pi} \) resp. the aggregated allocations \( \sum x_i \). Since \( \sum x_i \) remains a distance of \((x^*, -x^*)^T = \sqrt{2} \cdot x^* \) away from the market equilibrium, the market operator has no efficient method to compute neither \( x^* \) nor \( \Delta p \).

A closer look at the structure of the utility functions of class \( C_1 \) and \( C_2 \) reveals characteristics, which explain the slow convergence behavior of gradient based-price finding algorithms:

1) The partial derivatives are almost at all points zero and therefore don’t provide any information for the gradient-based search algorithm.

2) Due to the discontinuance of the utility functions there exist price values, at which devices change their consumption from their minimal period allocation to their maximal period allocation at once (in one iteration).

3) The homogenous structure of utility functions of different devices like shiftable loads and wind or solar farms lead to a joint commitment of multiple devices at certain price levels, thereby amplifying the discrete energy allocation steps in the market outcome. This is especially challenging because it negates the positive leveling effects of many smaller loads or generation units, which would otherwise contribute to a steadier commitment behavior.

4) Properties 2 and 3 lead to significant steps in the market outcome and doesn’t allow an iterative improvement of the solution, even when the market has a large number of market participants.

III. Related solution approaches

As already mentioned the problem of dealing with non-continuous non-convex demand and supply utility functions in the context of market-based optimization frameworks has already been discussed in the existing literature. Thus several heuristics have been developed to improve the stability and the
solution quality of the primal UCP. Here we want to give a brief review of the existing approaches and discuss their properties regarding the computational complexity, communication overhead, solution quality and practical feasibility. In [12], [13], [14], [15], the problem is solved by first computing an approximated solution for the dual problem using the classic Lagrange Relaxation (LR) approach with a fixed number of iterations and then improving the obtained solution by applying a heuristic search, which effects only a small subset of the committed units. The heuristic search is based on deterministic rules derived from expert knowledge to determine the units that have to be switched on or off in order to get a better approximation. While the results show an improvement of the obtained LR solution in the given test scenarios it is not clear if the heuristic can be generalized to other scenarios. Also if the market becomes very large e.g. millions of shiftable load appliances it is not clear how the heuristic search algorithm can be scaled up.

In contrast to the first approach [16], [17], [18] use the Bundle method to construct a cutting plane approximation $L^m(\lambda)$ of $L(\lambda)$ from the information collected in the $m$ previous iterations. The achieved approximation $\hat{L}^m(\lambda)$ can then be minimized in order to choose the next candidate for $\lambda$. While the cutting plane approximation is typically exposed to a linearization error, it prevents the iteration process from oscillating between multiple intermediate solutions. However in order to achieve good results the number of intermediate results used to construct the cutting planes has to be relatively high. The corresponding computational and communicational effort can be reduced by testing multiple candidates for $\lambda$ in parallel during each iteration. With respect to the problem of having many units committing or decommitting at fixed price levels (as indicated in section II-E) it is questionable if this method achieves better results than the classical Tatonnement process.

In [5] a combination of both methods is used to further improve the generated solution. First the dual problem is solved using bundle methods. Based on this solution a tabu search is applied to further improve the generated solution. A different approach for controlling shiftable loads in particular electric vehicles is proposed in [6]. In order to prevent a shiftable load to commit/ decommit its maximal power consumption in response to a favorite period price, a set of relative maximum demand limits is set for all load appliances. This enables the outer optimization problem to adjust the contribution of loads in each time period, without the need to change the corresponding $\lambda$ component. However different demand limits also effect the total energy generation costs [7]. Thus in addition to optimizing $\lambda$ the outer optimization problem has to choose an optimal demand limit, which doubles the number of decision variables. Besides this increase in complexity, many shiftable prosumers – including some electric vehicles with primitive charging controllers, non modulatable $\mu$CHP units, etc. – are true binary devices, which are not able to adjust their charging current relative to their maximum power.

IV. A RANDOMIZED PRICING SCHEME

Our solution is based on the hypothesis that given a large number of market participants, slightly different market prices for each market participant smooth out the discrete steps in the market outcome. This hypothesis is mainly motivated by the charging behavior of a group of electric vehicles with similar start and end periods. As described in section II-E a slight modification of a period price can cause the (de-)/ commitment of all electric vehicles at once. On the other hand if each electric vehicle would receive a slight offset in each price component equally distributed around the original price signal, the individual relative price thresholds that induce (de-) commitment would be slightly different as well. Thus the group behavior would result in a successive (de-) commitment of each electric vehicle based on its individual price offset. We think that other shiftable loads exhibit similar behavior. Especially wind- and solar farms, which can be modeled with equation (8) should behave in a similar way. The benefit of this method is that it requires only small changes in the pricing mechanism of the market operator – leaving the utility functions of the market participants untouched. Therefore it supports all kinds of prosumers regardless if they can modulate their power consumption or not.

A. Fairness

An important aspect that has to be considered in a market framework, in which each prosumer gets a slightly different price signal is the aspect of fairness. Since random price offsets may lead to a price signal, that is more or less preferable for a specific prosumer, a prosumer may gain a dis-/advantage in comparison to other prosumers. For example adding a positive price offset on the price signal increases the charging costs for an electric vehicle, while adding negative price offsets lower the charging costs. Thus random price offsets may discriminate a subset of prosumers. Beside the discrimination of prosumers random price offsets may also decrease the solution quality and increase the energy generation costs, because producers are not producing at the exact marginal costs of the whole market system. In order to minimize these effects we introduce two fairness-constraints:

- **Neutral price component offset distribution:** To limit the discrimination of a single consumer or producer over $k$ time periods, we require that the sum of all positive price component offsets should be equal to the sum of all negative component offsets. Thus a consumer/ producer, which allocates a fixed amount
of energy over \( k \) time periods will not be affected by the pricing scheme.

\[
\sum_{j=1}^{k} p_{\text{sample}}[j] - p[j] = 0 \tag{12}
\]

The equation describes a hyperplane in \( \mathbb{R}^k \) centered at the current price signal \( p \) with a normal vector of \((1, 1, \ldots, 1)^T\) (see figure 2a)

- **Limited magnitude:** The second equation restricts the deviation of the random price offsets to a given distance \( \epsilon \). In order to keep the individual price vectors close to the system marginal cost, \( \epsilon \) should be as small as possible. In section V we will investigate the impact of \( \epsilon \) on the convergence behavior as well as the solution quality.

\[
\sum_{j=1}^{k} (p_{\text{sample}}[j] - p[j])^2 \leq \epsilon^2 \tag{13}
\]

This equation represents the volume of a hypersphere (see figure 2b).

### B. Randomized sampling

Generating uniformly distributed samples can be a difficult task especially in higher dimensions. Fortunately in this case we can exploit some geometric properties to come up with an efficient sampling method. In order to generate uniformly distributed samples that satisfy equation (12) and (13) three steps are necessary. First a random point with a uniformly distributed direction from the origin is sampled. An efficient way of generating such a sample was proposed by Knuth in [19]. The algorithm exploits the fact, that the gaussian distribution is invariant against any affine transformation in particular against rotation. Thus a sample generated by choosing each component out of a gaussian distribution has a uniform distributed direction with respect to the origin. The second step is projecting the generated sample to the hyperplane defined by equation (12). At last the projected sample needs to be distributed uniformly within a radius of \( \epsilon \) to satisfy the fairness equation (13). The resulting algorithm is given by the following pseudo code:

**Require:** \( p \in \mathbb{R}^k \)

1: \textbf{for} \( i = 1 \ldots k \) \textbf{do}
2: \( s[i] \leftarrow \mathcal{N}(0, 1) \)
3: \textbf{end for}
4: \( d_{\text{plane}} \leftarrow \sum_{i=1}^{k} s[i] \)
5: \( s_{\text{proj}} \leftarrow s - (d_{\text{plane}}/k) \cdot (1, 1, \ldots, 1)^T \)
6: \( r_{\text{uniform}} \leftarrow k \cdot \sqrt{\text{uniform}(0, \epsilon)} \)
7: \( p_{\text{offset}} \leftarrow s_{\text{proj}} \cdot r_{\text{uniform}} / |s_{\text{proj}}| \)
8: \textbf{return} \( p + p_{\text{offset}} \)

Figure 3 depicts the result of 3000 randomly generated price offsets \((k = 3)\) using the proposed sampling method. As it can be observed the samples are uniformly distributed in the area defined by the fairness constraints.

![3000 random generated price offsets for k = 3](image1.png)

**Fig. 3.** 3000 random generated price offsets for \( k = 3 \)

### C. Distributed implementation

Two operations are necessary for the implementation of randomized price offsets. First, at the beginning of each period (e.g. in the first iteration of the NR-algorithm) a randomized price offset has to be generated for each prosumer using the algorithm presented in section IV-B. Second the system has to store a map of all prosumers and the corresponding price offsets in order to update new price vectors with the same price offsets in the following NR iterations. Typically ICT-architectures for market-based optimization frameworks utilize a hierarchical communication structure as depicted in figure 4 [20]. The structure reduces the communication complexity by aggregating the information at each price signal distributor node. The bottom layer of price signal distributors is suitable to store the required mapping information for the corresponding local subset of prosumer devices. Thereby the number of price signal distributors at the bottom layer may be extended to any arbitrary number in order to maintain scalability.

**Fig. 4.** ICT-architecture for generating and storing random price offsets

### D. Solution quality

As already mentioned in section IV-A a deviation from the system’s (aggregated) marginal costs may decrease the solution quality. However since \( \epsilon \) should be very small we think that the
positive effects of distributing binary commitment devices and shiftable prosumers outperform the negative effects. Especially a larger amount of market participants with small nominal power – in relation to the market volume – should improve the solution quality. Nevertheless the quality of the solution strongly depends on the market setup and has to be evaluated empirically.

V. Simulation Results

The simulation studies are focused on two aspects of the optimization framework. First we investigate the impact of epsilon ($\epsilon$) on the convergence behavior of the market mechanism and the solution quality measured as the difference to the theoretical optimal solution. In the second parameter study we vary the prosumer configurations and market size.

A. Basic market setup

For the market setup we consider the use case of a balancing responsible party (BRP), which wants to optimize its portfolio consisting of 53600 electric vehicles, 4000 wind turbines and a conventional power plant. In order to be able to make qualified statements about the experiment outcomes, we limit the optimization horizon to the night period from 22:00 to 4:00, which we consider as the major time interval for the recharging process. This interval will be divided into 24 – 15 min time periods. We further assume that the electric vehicles are all connected to the grid and may be freely shifted among the available time periods. The joint recharged energy adds up to $E_{\text{consume}} = 1.75$ GWh, while the maximal charging power for a single electric vehicle is limited to 11 kW per 15 min period. Do note that this setup induces highly non-convex non-continuous utility functions similar to the example of section II-E.

The wind turbines feed in an aggregated amount of 1.675 GWh following a wind-profile taken from the German TSO "50 Hertz" on 19.07.2013. Thereby each wind-turbine behaves according to equation (8), with a price threshold of 0. Thus the turbines will always feed in their maximal available power output as long as the market price remains positive. The generator on the other hand is modeled by equation (9) with $a = 2/3 \cdot 10^{-3}$. The quadratic cost term induces a penalty factor for not distributing the residual demand equally over all time periods. Also it lets us calculate the optimal residual demand distribution by $P_{\text{res}}[j] = 4 \cdot (1.75 - 1.675)/24 = 12500$ kW, which leads to a market clearing price of $P_{\text{clear}}[j] = P_{\text{res}}[j] \cdot 2a = 16.6$ Cent/kW and overall energy generation costs of $C_{\text{opt}} = 1.16$ Mill. Euro. Figure 5 summarizes the optimal energy allocations within the basic market setup.

B. Parameter study: epsilon factor

Figure 6 depicts the aggregated prosumer allocation of all market participants in each iteration for different static $\epsilon$ values as well as an adaptive $\epsilon$-heuristic. The adaptive $\epsilon$-heuristic starts with $\epsilon = 12$ dividing it by 2 each time the market is cleared until $\epsilon = 0.09375$.

It can be observed that higher values of $\epsilon$ lead to a faster convergence behavior. This can be explained by the larger radius around the market clearing price, for which the gradient based price finding method gets an improved feedback of the available shiftable prosumer devices. Furthermore the convergence behavior of the adaptive $\epsilon$-heuristic exhibits an interesting property: the distance to the market equilibrium of the iterations between two epsilon updates are almost identical. This indicates that the price gradients used to improve the solutions may also follow a certain pattern, which may be a good starting point for future improvements.

In addition to the convergence behavior figure 7 depicts the average solution quality of 100 test runs for different start price vectors chosen randomly from $[8, 22]^{24}$. The solution quality is measured by the opportunity costs of the BRP for not using an optimal algorithm. These costs consist of the opportunity costs for not reaching the optimal unit commitment and the difference in the cash-flow between consumers and producers. The latter cost element occurs because each prosumer gets slightly different prices. Clearly smaller $\epsilon$ values lead to a smaller difference, while too small values lead to slow convergence rates. The adaptive $\epsilon$-heuristic represents a compromise within this trade of by decreasing the difference to only 0.06% after 28 iterations!

C. Parameter study: consumer configurations

The last parameter study demonstrates the robustness of the adaptive $\epsilon$-heuristic against different consumer configurations and market sizes given in table I. In order to retain the total energy consumption and the supply configuration given in
section V-A $P_{\text{max}}$ is scaled accordingly. The results of scenario A, B, E (depicted in figure 8) indicate that the convergence behavior improves with the market size, which confirms our hypothesis stated in section IV. Besides the market size, different consumer configurations have influence on the convergence behavior (scenario B, C). This is because only 10% of the consumers influence $\approx 82\%$ of the market outcome. Thus more consumers are needed to smooth out the discrete commitment steps (scenario D).

VI. CONCLUSION

We presented a new method – based on randomized price offsets – to deal with non-convex, non-continuous supply and demand functions as they are typically induced by renewable energy resources and shiftable demand appliances. The method relies solely on small but fair alternations of the price signal for each prosumer leaving the utility functions untouched. Thus the method can be applied to any type of market participant, without taking extra means (like direct modification of the power output). Finally the results show that both the convergence behavior and solution quality improves with the market size, which is a nice feature considering the growing participation of smaller appliances on future energy markets.

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