Spinning and Non-spinning Reserve Allocation for Stochastic Security Constrained Unit Commitment

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Abstract—This paper proposes a formulation for a stochastic security constrained unit commitment, dispatch and reserve scheduling, considering N-1 security and non-spinning reserves. We employ a randomized optimization technique that is based on scenario generation of the uncertain variables, in this case the wind power, and offers probabilistic certificates regarding the robustness properties of the resulting solution. To demonstrate the efficacy of the proposed methodology, we carry out a simulation based study on the IEEE 30 bus system comparing the proposed formulation to other variants, as well as examine its behavior on networks with different congestion levels. In all case studies, we verify empirically, via Monte Carlo simulations, the probabilistic performance of our solution methodology.

Index Terms—Unit commitment, reserve scheduling, non-spinning reserves, chance constrained optimization, uncertain generation.

I. INTRODUCTION

Security in power systems has traditionally been associated with the ability to ensure the supply of power under contingencies. Maintaining a secure system is one of the primary roles of the Transmission System Operator (TSO), and requires careful planning and forecasting of uncertain elements. A common task in today’s market oriented environment is the day-ahead planning of unit commitment and reserve allocation. The concept of N-1 security is widespread in the context of power systems, and refers to the systems ability to survive any single component outage [1]. The amount and distribution of reserves often reflect this concept, such as ensuring that enough reserve capacity is available to replace the largest online unit. With the rapid growth of stochastic renewable energy sources such as wind and photovoltaic power, established methods for estimating security and quantifying reserves have been shown to be inadequate [2], [3]. In recent years, occasions of undesirable load shedding and outages due to large wind power forecast deviations have triggered research in the field of reserve scheduling for systems with high wind penetration, attempting to find a middle ground between security of supply and economic viability.

In [4], [5] the authors propose models for solving the unit commitment and reserve scheduling problems. They do not account for component outages and model the wind through scenarios without providing any guarantees for the robustness of the solution. In [6] a model is proposed for solving the security constrained unit commitment problem subject to nodal injection uncertainty, considering among others, transmission security constraints. The authors do not provide probabilistic certificates, but rather guarantee that the solution is robust against a bounded uncertainty. In the aforementioned references, no discussion regarding the deployment of the reserves in real time is provided.

In [7], [8] a novel security constrained reserve scheduling algorithm that accounts for real time deployment while providing a-priori probabilistic performance guarantees is proposed. In [9], the authors provide a stochastic unit commitment algorithm that employs the reserve deployment strategy proposed in [8]. The formulation provides probabilistic guarantees, but neither component outages nor non-spinning reserves are included. The aforementioned stochastic problems are solved using the methodology of [10] which is based on a mixture of randomized and robust optimization, allowing the authors to solve chance constrained programs that include integer decisions like those appearing in unit commitment problems.

In this paper we combine approaches from [8], [9] and formulate a tractable optimization program for the unit commitment and reserve scheduling problem, subject to stochastic wind infeed and conventional constraints, such as N-1 security, ramping limits, and generation minimum on and off times. Additionally, we formulate the deployment of non-spinning reserve power, where we indicate under what conditions offline generators should be turned on, and how the reserve power is allocated depending on the possible contingency and wind infeed.

In Section II we introduce the problem and explain several key elements of the proposed formulation. Section III provides the compact form of the optimization program and discusses how to deal with the chance constraint, while Section IV provides details regarding the problem formulation. Section V shows results of a simulation based study and a comparison with other reserve allocation methods. Finally, Section VI summarizes our approach, and provides directions for future research.

II. PROBLEM SET-UP

We consider a power system consisting of $N_G$ generators, $N_W$ wind parks, $N_b$ buses, $N_L$ loads and $N_L$ lines and let $N_t$ denote the horizon of the optimization problem. We use $T^{l}_{out} \in \mathbb{T}_{out}^{L} \in \mathbb{T}_{out}^{G} \in \mathbb{T}_{out}^{W}$ to denote the sets of indices corresponding to nodal...
to each type of outage, and let $\mathcal{I}$ be the union of these sets including the $0$ index that corresponds to the basecase of no outage.

We consider a DC power flow model [11] that makes several approximations: a) We assume all voltages to be $1$ p.u. b) Angle deviation is assumed small ($\sin \theta \approx \theta$). c) Line resistance is neglected. These approximations, even if they reduce the formulation precision, allow us to formulate the problem as a linear program and take advantage of established methods and tools to obtain relatively fast solutions for complex systems. We formulate line flows $P_l \in \mathbb{R}^{|\mathcal{N}|}$ as a function of the power injection vector $P_{\text{inj}} \in \mathbb{R}^{|\mathcal{N}|}$, i.e. $P_l = A_l P_{\text{inj}}$, where $A \in \mathbb{R}^{|\mathcal{N}| \times |\mathcal{N}|}$ depends on the network admittances and the superscript $i \in \mathcal{I}$ denotes the outage dependency of the variables [2]. The elements of the power injection vector are the sum of the production and consumption at each bus:

$$P_{\text{inj}}^i = C^i_G (P_G + R^i) + C^i_W P_W - C^i_L P_L$$

where $P_G \in \mathbb{R}^{|\mathcal{G}|}$ is the generator dispatch vector, $P_L \in \mathbb{R}^{|\mathcal{L}|}$ is the load consumption vector and $P_W \in \mathbb{R}^{|\mathcal{W}|}$ is the wind infeed. $C^i_G, C^i_W, C^i_L$ are matrices of appropriate dimensions that map the generators, wind parks and loads to buses, considering also the cases of outages. $R^i \in \mathbb{R}^{|\mathcal{G}|}$ is the generator reserve infeed.

Reserves are needed when there is a power mismatch, evident from frequency deviation. Automatic control loops have different response times and are often differentiated into primary and secondary control by the order in which they are deployed, with primary control being local to generators, and secondary control being centrally deployed by the TSO. Prolonged outages or extreme forecast deviations may require manual intervention, most often referred to as tertiary control. The formulation described in this paper does not distinguish between secondary and tertiary control, and could equally apply to both, but we assume that the system is in steady state and that no other controllers are affecting production or consumption.

For our formulation we account for power mismatch due to wind infeed deviation from, or the outage of a line, load or a generator. We define the power mismatch (or production excess) vector $P_{m}^i$ for all indices $i \in \mathcal{I}$:

$$P_{m}^i = \sum_{k \in \mathcal{W}^i \cup \mathcal{K}^i} (P_{W}^k - P_{W}^{k/f})$$

where the first line is the sum of the wind infeed deviation and the second line shows the power mismatch contribution of outaged wind parks, generators and loads, where $b_W$, $b_G$ and $b_L$ are of appropriate dimensions and are zero vectors for no outage, or contain a single “1” in the position of a relevant outage. In the event of a wind park outage, the power mismatch should include the actual wind infeed but omit the forecasted infeed. This can be accomplished through the $K^i$ set, which is a singleton for wind park outages, containing the outaged wind park index, while being the empty set for other outages.

The distribution of the power mismatch between generators is predetermined as part of the preventive day ahead dispatch, distributed between generators using weight vectors, typically referred to as distribution vectors [2], [7]. In the simplest case, we define a single vector which multiplied by the mismatch indicates how much each generators set-point should be adjusted. In the case of a generator outage, that unit can not participate in supplying reserves, so for the outage of each generator we must have a separate vector with zero in the appropriate element. For power systems equipped with measurement devices that can detect load and line outages and communicate those to the TSO in a fast and reliable manner, we employ a corrective security control strategy by having more distribution vectors, depending on the outage, offering more degrees of freedom with the potential of decreasing reserve cost.

We also make use of offset vectors, that depending on the outage and wind infeed, shift the generation of individual generators by a fixed amount without affecting the total production. This is useful, for example in the case of a line outage, generation can be shifted around the system in order to reduce the impact of the outage on transmission constraints. Another example is the case of a generator outage, where the offset can be used to bring otherwise offline generators online and into their operating region, enabling effective use of non-spinning reserves.

The reserve infeed is modeled similar to the convex formulation in [7] and we define the following:

- The distribution vectors $d_{\text{up}}^i$ and $d_{\text{down}}^i$ for $i \in \mathcal{I}$.
- The offset vectors $P_{\text{up,offset}}^i$ for $i \in \mathcal{I} \setminus \{0\}$ and $P_{\text{down,offset}}^i$ for $i \in \mathcal{I} \setminus \{0\}$.
- The compensation vectors $P_{\text{g,comp}}^i$ for $i \in \mathcal{I} \setminus \{0\}$ that compensate for outaged generation.

We also define the sum of the wind infeed deviation:

$$\Delta P_W = \sum_{i=1}^{N_W} (P_{W}^i - P_{W}^{i/f})$$

The reserve infeed, $R^i$ as seen in (1) is finally defined as:

$$R^i = \begin{cases} 
  d_{\text{up}}^i \max_{+}(-P_{W}^i) - d_{\text{down}}^i \max_{+}(P_{m}^i) + P_{\text{up,offset}}^i B(-P_{m}^i) + P_{\text{down,offset}}^i B(P_{m}^i) \\
  P_{\text{g,comp}}^i B(\Delta P_W) + P_{\text{g,comp}}^i B(\Delta P_W)
\end{cases}$$

where the element-wise (step) function:

$$B(x) = \text{sgn}(\max(x,0)) = \begin{cases} 
  1 & x > 0 \\
  0 & x \leq 0
\end{cases}$$

In the generator outage case we follow the approach of [7] to avoid the bilinear term that would otherwise arise from
the multiplication of \( d \) and \( P_m \), and instead multiply the distribution vectors with the wind deviation, and use the compensation vector \( P_{g,\text{comp}} \) to make up for the missing generation by enforcing \( \sum P_{g,\text{comp}} = P_G \). Further, we omit the \( P_{d,\text{offset}} \) vector for the generator outage cases since the vector \( P_{g,\text{comp}} \) behaves in a similar fashion and makes the explicit offset vector redundant. The reserve infeed definition above is no longer a simple product of distribution vectors and the power mismatch, and requires appropriate measurements of wind infeed and outages and their communication to the controller.

Example: An example of reserve deployment can be seen in Fig. 1 where we have a three generator system with a wind park. The figure depicts the situation when generator 2 is out of service (while scheduled to generate 15 MW) and generator 3 is offline but can be brought online to supply reserves. The points on the vertical axis denote the scheduled generation while other points indicate generator set-points during the outage. The dotted lines show generator limits, the dashed lines between the outage. The points on the vertical axis denote the scheduled generation of service (while scheduled to generate 10 MW) and generator set-points during the outage of generator 2 scheduled to produce 15 MW during the outage of generator 2. The generator 2 goes down, one of two things happen:

a) If the wind infeed is more than expected (\( \Delta P_W > 0 \)), we shift the set-point of generator 1 to compensate fully for the missing generation (\( P_{g,\text{comp}} = [15, 0, 0] \)) and the reserve infeed increases or decreases opposing the deviation in wind infeed (\( d_{\text{up}} = [1, 0, 0] \)). Generator 3 is kept offline.

b) If the wind infeed is less than expected (\( \Delta P_W < 0 \)), generator one is not capable of compensating for the missing production. We turn on generator three (\( P_{\text{up,offset}} = [-15, 0, +15] \)) and then split the wind deviation between the generators (\( d_{\text{up}} = [0.5, 0, 0.5] \)).

III. DEALING WITH THE UNCERTAINTY

Unit commitment problems are often described by optimization programs and in this paper we assume a problem with a quadratic (or linear) objective, linear constraints and a decision variable vector comprised of real and/or binary variables. The problem is commonly expressed as:

\[
\begin{array}{ll}
\text{minimize} & J(x) \\
\text{subject to} & Ax \geq b \\
& x \in X
\end{array}
\]

where \( J(x) = x^T Q x + c^T x \). The domain of the individual elements of \( x \) is either \( \mathbb{R} \) or binary \((0, 1)\), with further restrictions formulated by \( A \). Most variables in the conventional unit commitment problem are continuous, with the notable exception of the variables indicating the on/off status of the generators.

The stochasticity of the wind infeed affects the problem formulation in the sense that the matrix \( A \) of problem (4) must account for the uncertainty. We introduce \( \delta \in \Delta \subseteq \mathbb{R}^{3N} \) as a vector of the deviation of wind infeed from the wind forecast, where \( \Delta \) signifies the uncertainty set. The structure of our problem is such that the wind error \( \delta \) is only multiplied with continuous variables, we therefore reformulate the problem as:

\[
\begin{array}{ll}
\text{minimize} & J_x(x) + J_u(u) \\
\text{s.t.} & \mathbb{P}(\delta \in \Delta | A(\delta) x + B u + c(\delta) \leq 0) \geq 1 - \varepsilon \\
& x \in X; u \in \{0, 1\}^n
\end{array}
\]

where \( x \) is a vector of all real decision variables, \( u \) is a vector of all binary decision variables, and \( A \) and \( c \) are piecewise affine functions of the uncertainty vector. The chance constraint \( \mathbb{P}(\cdot) \) indicates that with a probability greater than \( 1 - \varepsilon \) the enclosed constraints are valid.

To solve the chance constrained problem we use the probabilistically robust design of [10]. This is a randomized optimization based technique that does not impose any assumptions on the probability distribution of the uncertainty or the structure of the underlying constraints while providing a-priori guarantees regarding the probability of constraint satisfaction.

This technique is inspired by the so called scenario approach that is proposed in [12]. Based on the scenario approach, the chance constraint is transformed to a finite number of hard constraints and is satisfied with a certain confidence. However, it requires convexity of the underlying constraint functions, with respect to the decision variables. Therefore, it cannot be used for the stochastic unit commitment problem since the convexity requirement is not satisfied due to the presence of binary decision variables. However, since the probabilistically robust design does not require any particular structure of the
constraints, we can solve the stochastic UC problem while getting a-priori the robust optimization performance guarantees.

This technique includes two steps. In the first step, the scenario approach [12] is used to determine, with a confidence of at least $1 - \beta$, the minimum volume set that contains at least $1 - \varepsilon$ probability mass of the uncertainty. Details on how to determine such a set can be found in [10], [8]. To compute this set (denoted here by $D$) with these properties, following [13], a certain number of scenarios need to be generated, given by:

$$N \geq \frac{1}{\varepsilon} \left( \frac{1}{\beta} + 2N_d - 1 \right)$$

where $N$ is the number of scenarios, $\varepsilon$ and $\beta$ are the violation and confidence parameters, and $N_d = N_W N_t$ for guarantees over $N_t$ hours, or $N_d = N_W$ for individual hours.

In the second step, we use the probabilistically computed set $D$ and formulate a robust problem where the uncertainty is confined in this set. In this way, the chance constraint (8) is substituted by the following robust constraint:

$$Fx + f + H \delta \leq 0, \text{ for all } \delta \in D$$

The interpretation of (7) is that the constraint should be satisfied for all values of $\delta \in D$. Following [10], any feasible solution of this robust problem is feasible for the chance constraint (8) with a probability of at least $1 - \beta$. This guarantee arises from the fact that $D$ is chosen in a probabilistic way. To solve the resulting robust program the reader is referred to [14], [10].

IV. PROBLEM FORMULATION

For the formulation we define $\delta = P_W - P_W^f \in \mathbb{R}^{N_W}$ and introduce several variables:

- $R_{up}, R_{down}, R_{standby} \in \mathbb{R}^{N_G}$ are the reserve vectors, indicating how much up, down and non-spinning reserves were allocated to (bought from) each generator.
- $u \in \{0,1\}^{N_G}$ is a binary vector indicating whether generators are on or off for a particular hour.
- $s_{up}, s_{down} \in \{0,1\}^{N_G}$ for $i \in \mathcal{I}$ are binary vectors indicating if offline generators should be brought online for up/down spinning regulation. $\delta \in \mathbb{R}^{N_G}$ is a vector indicating whether a generator is available for non-spinning reserves for any outage, and can be left as a continuous variable even though it will only take binary values.
- $z \in \mathbb{R}^{N_G}$ is an auxiliary variable indicating whether a generator unit was turned on between two consecutive hours. Similar to $\delta$ it can be formulated as a continuous variable although it will only take binary values.

We stack all the real decision variable vectors corresponding to one time instance $t$ in the vector:

$$x_t = [P_{G,t}, R_{up,t}, R_{down,t}, R_{standby,t}, d_{up,t}^r, d_{down,t}^r, P_{comp,t}^r, P_{up,offset,t}^r, P_{down,offset,t}^r, z_t, \delta_t] \in \mathbb{R}^{N_G(8+4(N_L+N_W+N_G))}$$

and the binaries in the vector:

$$y_t = [u_t, s_{up,t}^r, s_{down,t}^r] \in \{0,1\}^{N_G(1+2(N_L+N_W+N_G))}$$

where $* \in \mathbb{R}^{N_G}$ signifies all possible values of $i$, as appropriate. The optimization problem is given by:

$$\min_{\{x_t, y_t\}_{t=1}^{N_t}} \sum_{t=1}^{N_t} \left( P_{G,t}^2 + c_{e} \right) \left( P_{G,t} + c_0 u_t \right) + c_{up} R_{up,t} + c_{standby} R_{standby,t} + c_{down} R_{down,t} + c_{on} z_t$$

where the undefined $c$ vectors represent various production costs and $[c]$ is a square matrix with $c$ on the diagonal. The problem is subject to many constraints enclosed in the $A(\delta), B$ and $c(\delta)$ matrices. We first list the constraints that are independent of the wind infeed deviation $\delta$:

1) Power balance constraints; scheduled production and consumption must be equal for all hours. For $t = 1, \ldots, N_t$:  

$$1 \times N_t \left( C_G P_{G,t} + C_W P_{W,t} - C_L P_{L,t} \right) = 0$$

where $P_{W,t} \in \mathbb{R}^{N_W}$ is the forecasted wind infeed.

2) Generators must operate within their generation limits:

$$P_G \circ u_t \leq P_{G,t} \leq \overline{P}_G \circ u_t \quad \text{for } t = 1, \ldots, N_t$$

where $\circ$ denotes element-wise multiplication and $P_G, \overline{P}_G \in \mathbb{R}^{N_G}$ are vectors of the generators minimum and maximum production. By multiplying $u$ with the generation limits we ensure that offline generators have zero scheduled production.

3) Distribution vectors must sum to one, for all $i \in \mathcal{I}$ and $t = 1, \ldots, 24$:

$$1 d_{up,t}^i = 1 \quad 1 d_{down,t}^i = 1$$

4) Generator compensation vectors must sum to the missing generation, for all $i \in \mathcal{I}_{out}$ and $t = 1, \ldots, N_t$:

$$1 P_{G,comp,t}^i = P_{G,t}^i$$

5) The offset vectors must sum to zero, so they don’t affect the power balance. For all $t = 1, \ldots, N_t$:

$$1 P_{offset,up,t}^i = 0 \quad \text{for all } i \in \mathcal{I} \setminus \{0\}$$

6) Generators can either be online, on standby, or off:

$$\delta_t \leq (1 - u_t) \quad \text{for } t = 1, \ldots, N_t$$

7) Outage specific standby vectors must be less than the overall standby vector $\delta_t$, for $t = 1, \ldots, N_t$:

$$s_{up,t}^i \leq \delta_t, \quad s_{down,t}^i \leq \delta_t \quad \text{for all } i \in \mathcal{I} \setminus \{0\}$$

8) Generator inter-hour ramping constraints are represented for each $t = 2, \ldots, N_t$:

$$P_{G,t} - P_{G,t-1} \leq RU$$

$$P_{G,t-1} - P_{G,t} \leq RD$$

where $RU$ and $RD$ denote up and down ramping limits.
9) Reserves must be within ramping limits, for $t = 1, \ldots, N_t$:

\[
0 \leq R_{\text{up},t} \leq RRU \circ u_t
\]
\[
0 \leq R_{\text{down},t} \leq RRD \circ u_t
\]
\[
0 \leq R_{\text{standby},t} \leq RRU \circ \delta_t
\]

where $RRU$ and $RRD$ denote up and down reserve ramp limits.

10) To model the minimum time a generator needs to be either online or offline, we follow the approach of [5] using the auxiliary variable $z_t$:

\[
\begin{align*}
\ z_1 & = 0 \\
\ z_t & \geq u_t - u_{t-1} \quad \text{for } t = 2, \ldots, N_t \\
\ \sum_{\tau=t-T_U+1}^{t+T_D} z_{\tau} & \leq u_t \quad \text{for } t = T_U, \ldots, N_t \\
\ \sum_{\tau=t+1}^{t+T_D} z_{\tau} & \leq (1 - u_t) \quad \text{for } t = 1, \ldots, (N_t - T_D)
\end{align*}
\]

where $T_U$ and $T_D$ indicate the minimum on and off times for each generator. If $T_U$ and $T_D$ are not the same across all generators, the last two constraints above need to be implemented individually for each generation unit.

Other constraints of the problem involve the wind infeed deviation $\delta$ and are therefore represented by a chance constraint:

\[
\Pr(\delta \in \Delta | -P_t \leq A_i^i P^i_{\text{inj},t} (\delta) \leq P_t, (u_t + s^i_t) \circ P^i_G \leq P^i_{\text{up},t} + R^i_u (\delta) \leq (u_t + s^i_t) \circ P^{-i}_G, -R_{\text{down},t} \leq R^i_i (\delta) \leq R^i_{\text{up},t} + R_{\text{standby},t}, \forall i \in I) \geq 1 - \epsilon \quad \text{for } t = 1, \ldots, N_t
\]

where $\circ$ again denotes element-wise multiplication. As discussed in the last section, we transform this chance constraint into a fixed number of hard constraints, allowing us to formulate and solve the problem using standard approaches. Looking closer at the constraints enclosed in (8), we have:

1) The transmission line flow constraints:

\[
-\mathcal{P}_t \leq A^i_{ij} P^i_{\text{inj},t} \leq \mathcal{P}_t
\]

where $A^i_{ij}$ as explained earlier depends on network admittances and each line; every line results in a different $A$. The injection vector is a then a function of the wind infeed and outages as shown in (1).

2) We have the generator production limits, here accounting for the added reserve infeed and the standby status of the generator:

\[
(u_t + s^i_t) \circ P^i_G \leq P^i_{\text{up},t} + R^i_u \leq (u_t + s^i_t) \circ P^{-i}_G
\]

where $R^i_u$ is the reserve infeed as defined in (3). The standby vectors $s^i_t$ can be seen as a function of the wind infeed and power mismatch; there is a standby vector corresponding to each distribution vector:

\[
s^i_t = \begin{cases} 
  s^i_{\text{up},t} & \text{for } i \in I \setminus \mathcal{I}^G_{\text{out}} \text{ and } P^i_m < 0 \\
  s^i_{\text{down},t} & \text{for } i \in I \setminus \mathcal{I}^G_{\text{out}} \text{ and } P^i_m > 0 \\
  s^i_{\text{up},t} & \text{for } i \in \mathcal{I}^G_{\text{out}} \text{ and } \Delta P_W < 0 \\
  s^i_{\text{down},t} & \text{for } i \in \mathcal{I}^G_{\text{out}} \text{ and } \Delta P_W > 0
\end{cases}
\]

In practice we can fix some of the standby vectors to zero in order to reduce problem complexity. As an example in the case of excess production ($P_m > 0$) a standby generator is not useful.

3) Finally we have the reserve constraint:

\[
-R_{\text{down},t} \leq R^i_t \leq R_{\text{up},t} + R_{\text{standby},t}
\]

where we simply use the decision variable vectors $R_{\text{down}}$, $R_{\text{up}}$ and $R_{\text{standby}}$ to “contain” the most extreme reserve infeeds, determining the amount of reserves that need to be bought from each generator.

Since the reserve infeed is linear with respect to the power mismatch (for $i \in I \setminus \mathcal{I}^G_{\text{out}}$) and the wind infeed deviation (for $i \in \mathcal{I}^G_{\text{out}}$) we can reduce the number of scenarios that need consideration drastically. For example, if we look at the line outage constraints for a system with $N_W = 1$, as long as the solution is valid for no wind deviation and both up and down offset vectors, as well as for the minimum and maximum wind infeed scenarios, we can be certain that the solution is valid for all the wind scenarios in between the minimum and maximum. Similar derivations can be made for other outage constraints and more wind parks.

V. Simulations / Results

For the simulations we used a modified version of the 30 bus IEEE system (case30) from MATPOWER [15], which is composed of 6 generators, 41 line and 20 loads. Generator price data was altered and missing numbers such as reserve and non-spinning reserve costs were added in an attempt to resemble more realistic generator figures. Additional data was also included, such as generator minimum on and off times and maximum ramp rates. The day-ahead load curve was generated by arbitrary scaling of individual loads, and can be seen in Fig. 2.

In this study we considered a wind park located on bus 22, however, our analysis captures cases of wind infeed at multiple buses as well. We used normalized hourly measured wind power data, both forecasts and actual values, for the total wind power infeed of Germany over the period 2006-2011 to build a model for the possible wind power realizations (scenarios) based on [16]. For all simulations we had $\epsilon = 0.1$, $\beta = 10^{-5}$ and $N_d = N_W = 1$, giving us $N \geq 198$, for solving the optimization program we therefore generated a set of 200 scenarios for a period of 24 hours.

Programming was done in MATLAB using MATPOWER [15] for case data and power system specific functions. The optimization program was formulated using YALMIP [17] and solved using the commercial CPLEX [18] solver.
A sample solution of generator dispatch can be seen in Fig. 3 where we see the scheduled dispatch for all the generators in the system, along with the specific dispatch and reserve capacity (indicated by error bars) of generators three and four. We notice that generator six is offline for the entire day, reducing the fixed cost associated with keeping all the generators online. All the generators participate in offering reserves, even generator six which offers non-spinning reserves during peak hours. Generator three, depicted in Fig. 3(b), is connected to the same bus as the wind park, making it an ideal candidate to balance against wind infeed fluctuations without being obstructed by transmission constraints. Generator four (Fig. 3(c)) also participates with non-spinning reserves during hours of high demand, and is brought online during the peak hours.

We compared our final unified formulation to other reserve scheduling approaches, more specifically we ran three types of simulations:

1) The full formulation as explained in previous sections, we consider wind uncertainty and include non-spinning reserves and N-1 security.

2) The full formulation but with the non-spinning reserves disabled.

3) A rule-based approach for the day ahead unit commitment and reserve scheduling where the wind park infeed was expected to follow the forecast. Reserve power was determined from the rule of thumb that enough reserves should be available to replace the largest generator and load, with an additional 20% safety margin. Transmission constraints were considered when determining the dispatch, but ignored for the reserve infeed, and the solution does not consider N-1 security. This might be a typical solution for a simple system with little wind infeed and abundant transmission capacity.

We ran the simulations for different transmission capacities in order to determine its influence on price and security.

To verify the probabilistic performance of the solutions we carried out Monte Carlo simulations with 10,000 scenarios, roughly corresponding to \( \varepsilon = 0.01 \). We considered a scenario violated if the solution was not N-1 secure for a particular wind infeed, even though the solution may be acceptable for the base case and a subset of outages. The comparison can be seen in Fig. 4, revealing that not surprisingly for methods one and two we only see a small amount of scenario violations that are independent of transmission capacity. This is expected, as we guarantee the solution to be valid for a certain range of wind deviation, and reduced transmission capacity simply results in a more expensive allocation of reserves. The rule based method, while working well for the high capacity network, does not guarantee N-1 security for most or any of the reduced capacity cases. This is also reasonable; a lightly loaded network can allocate it’s reserves to generators far from potential shortages, while a congested network can not safely move reserve capacity from the cheapest generators to where it is needed.

The price comparison, seen in Fig. 5 shows a fixed price for the rule-based approach while the unified-approach becomes more expensive with reduced capacity. This is due to the fact that the rule based approach ignores transmission constraints when allocating the reserves, making it largely independent of the transmission capacity, while the unified approach, with
reduced transmission capacity, moves the reserves to better located but potentially more expensive generators.

The downside of the approach formulated in this paper, compared to simpler approaches such as the rule-based method demonstrated above, lies first and foremost in the optimization program complexity. The number of binary and continuous variables associated with our approach quickly becomes large, as it grows with the square number of generators and problem portionality to the number of hours. Exact measurements and predictions for solving time are difficult for mixed-integer programs associated costs, can offer cheaper solutions than otherwise feasible. Comparing with other methods, the precise estimation of required reserve power results in comparable or cheaper solutions, while offering enhanced security, even for congested networks.

Current work concentrates on alleviating the computational complexity of the proposed algorithm by exploiting decomposition techniques and/or distributed optimization.

VI. CONCLUDING REMARKS

In this paper we formulated a security constrained optimal power flow that accounts for unit commitment, dispatch and reserve power determination and allocation, including non-spinning reserves and stochastic wind infeed while providing probabilistic guarantees. The performance of the proposed approach was demonstrated by means of Monte Carlo simulations. It was shown that the stochastic program is solved efficiently satisfying the desired probabilistic guarantees. Moreover, we see that non-spinning reserves, depending on the associated costs, can offer cheaper solutions than otherwise feasible. Comparing with other methods, the precise estimation of required reserve power results in comparable or cheaper solutions, while offering enhanced security, even for congested networks.

REFERENCES
