A Linear Decision Rule Approach for Robust Unit Commitment Considering Wind Power Generation

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Abstract—This paper proposes a robust optimization formulation to solve unit commitment (UC) problems under wind energy uncertainties. Unlike the conventional stochastic programming or chance-constrained methods, this robust approach does not require information on the exact distribution of wind power. Instead, it protects the system against load loss under all possible uncertainties. Unlike the conventional stochastic programming or chance-constrained methods, this robust approach does not need information on the exact distribution of wind power. Instead, it protects the system against load loss under all possible uncertainties.

Index Terms—Unit Commitment, Adjustable Robust Optimization, Uncertainty Set, Linear Decision Rule.

I. NOMENCLATURE

A. Indices

- $b$: Indices of transmission lines
- $i$: Indices of recourse decisions
- $j$: Indices of linear blocks that approximate quadratic power production cost functions
- $k$: Indices of wind power generation sources
- $l$: Indices of the constraints of recourse decisions
- $m$: Indices of buses
- $n$: Indices of generators
- $s$: Indices of uncertainty variables
- $t, \tau$: Indices of time steps

B. Sets and Functions

- $B$: Set of all transmission lines
- $C_{nt}(\cdot)$: Generation cost function of unit $n$, at time step $t$
- $I$: Set of all recourse decision variables
- $J$: Set of linear functions that approximate quadratic power production cost functions
- $K$: Set of all wind power generation sources
- $K_m$: Set of wind power generation sources at bus $m$
- $\mathcal{L}$: Set of all constraints of recourse decisions
- $M$: Set of all buses
- $N$: Set of all generators
- $N_m$: Set of generators at bus $m$

C. Constants

- $A_n^j$: Linear term of the $j$th segment of the linear piecewise generation cost function ($$/\text{MWh})$
- $B_n^j$: Constant term of the $j$th segment of the linear piecewise generation cost function ($$)
- $C_n^{RD}$: Cost of downward reserve from unit $n$ ($$/\text{MWh})$
- $C_n^{BU}$: Cost of upward reserve from unit $n$ ($$/\text{MWh})$
- $C_n^S$: Cost of starting up unit $n$ once ($$)
- $D_{mt}$: Electricity demand of bus $m$ at time step $t$ (MW)
- $F_b$: Capacity of transmission line $b$ (MW)
- $G_n^{\max}$: Maximum capacity of generator $n$ (MW)
- $G_n^{\min}$: Minimum capacity of generator $n$ (MW)
- $I_n^u$: Minimum number of time steps that unit $n$ should be initially on line
- $I_n^d$: Minimum number of time steps that unit $n$ should be initially off line
- $K_{bm}$: Line flow distribution factor for transmission line $b$ at bus $m$
- $RD_n$: Ramp-down limit of unit $n$ (MW/h)
- $RU_n$: Ramp-up limit of unit $n$ (MW/h)
- $T_n^{u}$: Minimum up time of unit $n$
- $T_n^{d}$: Minimum down time of unit $n$
- $W_{kt}$: The expected value of wind power generation from wind farm $k$ at time step $t$ (MW)
- $W_{kt}^+$: Maximum upper deviation of wind power generation from wind farm $k$ at time step $t$ (MW)
- $W_{kt}^-$: Maximum lower deviation of wind power generation from wind farm $k$ at time step $t$ (MW)
- $\Gamma$: Uncertainty budget of random wind power output

D. Random Variables and Uncertainty Factors

- $w_{kt}(z)$: Random wind power generation (MW)
- $z_{kt}^{(+)}$: Uncertainty factor that indicates the upper deviation of wind power generation from its expected
E. Decision Variables

$z_{kt}$: Uncertainty factor that indicates the lower deviation of wind power generation from its expected value.

$g^N_{nt}$: Dispatch decision in terms of the generation output from generator $n$ at time step $t$, in the nominal scenario (MW).

$g^Z_{nt}(z)$: Redispacth decision in terms of the generation output from generator $n$ at time step $t$, under the realization of wind power generation $z$ (MW).

$q^N_{kt}$: Dispatch decision in terms of the wind power generation from wind farm $k$ at time step $t$, in the nominal scenario (MW).

$q^Z_{kt}(z)$: Redispatch decision in terms of the wind power generation from wind farm $k$ at time step $t$, under the realization of wind power generation $z$ (MW).

$r^U_{nt}$: Upward reserve provided by generator $n$, at time step $t$ (MW).

$r^D_{nt}$: Downward reserve provided by generator $n$, at time step $t$ (MW).

$u_{nt}$: Binary variables indicating the on/off status of unit $n$ at time step $t$.

II. INTRODUCTION

Unit commitment (UC) determines the optimal schedule of unit status over a given operation horizon. One of the main challenges in modeling UC problems comes from the influence of system uncertainties, especially the uncertainty of renewable energy sources, such as wind power generation. In recent years, the penetration of wind power keeps increasing rapidly worldwide as an effort to reduce the emission of greenhouse gases and to provide clean energy. However, it is also found that wind power is highly uncertain and intermittent, and cannot be dispatched in the classical sense [1]. Conventionally, the impact of system uncertainties is dealt with by applying deterministic reserve criteria that require extra generation capacity to be at least the largest on-line unit or a specific percentage of the peak load. Though easy to implement, these methods may not be able to provide satisfactory solutions because the influence of uncertainty is not properly modeled, especially in the presence of highly volatile wind power generation.

Among all approaches that attempt to explicitly incorporate uncertainty wind model into UC problems, the stochastic programming [2] might be the most popular one during the last decade. This approach considers the expected performance over a number of discrete scenarios that are constructed based on the actual probability distributions. It has been shown in [3], [4], [5] that stochastic UC formulations have better reliability and economic performance than their deterministic counterparts when dealing with uncertain wind power generation. However, stochastic programming is usually computationally challenging due to the large problem dimensions, as a great number of scenarios are required to represent the possibility of uncertain variables. Besides, it is also difficult to identify the exact distribution of uncertain wind power generation, especially when multiple wind farms, or multiple sources of renewable energy generation are considered.

As an alternative method for optimization under uncertainty, robust optimization [6] looks for solutions that protect systems against all realizations within a selected uncertainty set. Such an uncertainty set can be constructed without knowing the exact probability distribution of uncertain variables. This feature is particularly suitable for modeling wind power generation, which is usually very difficult to forecast. Efficient solution algorithms have been developed by Ben-Tal and Nemirovski to solve robust counterparts considering ellipsoidal uncertainty sets [7], and research by Bertsimas and Sim solves a polyhedral uncertainty set without increasing the computational complexity of the original problem [8], [9]. It has also been shown in these studies that the level of risk-aversion can be well controlled by applying different values of the uncertainty set parameters.

In order to solve multi-stage decision-making problems, the idea of adjustable robust optimization introduced by Ben-Tal et al. greatly extended the scope of robust optimization [10]. In the framework of adjustable robust optimization, some decisions are allowed to be made after the realization of system uncertainties, thus leading to better objective value and less conservative solutions [11], [12]. However, this benefit is achieved at higher computational burden, as the adjustable robust counterpart is usually NP-hard to solve. This difficulty can be addressed by using affine adjustable robust counterparts, a scheme that restricts recourse decisions to be linear affine functions of uncertain parameters [10], or by adopting decomposition algorithms [13].

In recent years, robust optimization techniques have been applied to solve UC scheduling problems. For example, an $n - K$ security criterion is proposed in [14], [15] to address the unit outage contingencies in daily power system operation. References [16], [17], [18], [19] modeled the impact of system uncertainties by the adjustable two-stage robust optimization framework, and [20] incorporated the both unit outages and load uncertainty into an adjustable robust optimization formulation, so that the system is protected against the combined impact of multiple sources of uncertainty. These two-stage robust UC formulations are commonly solved by decomposition algorithms, which, however, cannot guarantee the computational tractability.

In this paper, we develop a new adjustable robust optimization formulation that solves the joint energy and reserve dispatch problem [21], [22] under uncertain wind power generation. The proposed method protects power systems against all possible realizations of wind power generation in a preselected uncertainty set. It guarantees that there are always feasible redispatch solutions to supply the demand after the realization of wind power generation. Unlike the previous studies that utilizing decomposition algorithms to solve the adjustable robust counterpart, we apply the linear decision rule [23] to approximate the wait-and-see redispatch decisions, so that this
robust UC formulation remains computationally tractable. The remaining part of this paper is organized as follows. In the next section, the uncertainty set model for the random wind power generation is discussed, followed by the detailed robust formulation of the UC problem, as well as the approximation using linear decision rule techniques. Case studies based on the IEEE RTS-1996 [24] are presented in section IV to demonstrate the effectiveness of the proposed uncertainty model. Concluding remarks are provided in the final section.

III. FORMULATION

This paper tries to solve UC problems in a joint energy and reserve market environment. We mainly focus on the uncertainty of wind power generation, because it is more difficult to predict the output of wind power, and the forecast error of wind power generation is normally much larger than the forecast error of load [25]. It is assumed that there are several wind power sources, located at different buses of the system, and the influence of uncertain wind power generation is captured by a DC flow model.

A. Uncertainty Set for the Random Forecasting Error of Wind Power Generation

The proposed robust optimization approach minimizes the total cost of generation and reserve based on offers, while ensuring that the demand can be supplied under all wind power realizations over an uncertainty set. The uncertain wind power generation is expressed as function (1).

$$w_{kt}(z) = \tilde{W}_{kt} + W^+_{kt}(z^+) - W^-_{kt}(z^-)$$  \hspace{1cm} (1)$$

where the randomness of the wind power forecasting error is represented by the uncertainty factor $z = (z^+, z^-)$. The uncertainty factors are constrained within an uncertainty set, denoted by $\mathcal{Z}$ and expressed as follows.

$$\mathcal{Z} = \left\{ z : \begin{array}{l} 0 \leq z^+_{kt} \leq 1 \\
0 \leq z^-_{kt} \leq 1, \forall k \in K, \forall t \in T \\
\sum_{k \in K} (z^+_{kt} + z^-_{kt}) \leq \Gamma, \forall t \in T \end{array} \right\}$$ \hspace{1cm} (2)$$

This uncertainty sets suggests that the expectation of wind power output is $\tilde{W}_{kt}$, and wind power output is between $W^-_{kt} - W^+_{kt}$ and $W^-_{kt} + W^+_{kt}$. The overall level of uncertainty at each time step is controlled by the budget of uncertainty parameter $\Gamma$. For larger values of $\Gamma$, more realizations of wind power generation are included into the uncertainty set, leading to more conservative decisions. In contrary, the level of conservatism is reduced if smaller values of $\Gamma$ are selected.

B. The Adjustable Robust Counterpart of the Unit Commitment with Uncertain Wind Power Generation

Let $z = (z^+, z^-)$ indicate the realization of wind power generation, the formulation of the UC under uncertain wind power generation based on an adjustable robust optimization framework is given below.

$$\min \sum_{n \in N} \sum_{t \in T} \left\{ S^G_{nt}(u) + C^G_n(g^N_{nt}, u_{nt}) \\
+ C^{RU}_{nt} r^U_{nt} + C^{RD}_{nt} r^D_{nt} \right\}$$ \hspace{1cm} (3)$$

s.t. \quad S^G_{nt}(u) = \max \left\{ 0, C^G_n(u_{nt} - u_{nt(t-1)}) \right\}, \forall n \in N, \forall t \in T$$ \hspace{1cm} (4)$$

$$C^G_n(g^N_{nt}, u_{nt}) = \max_{j \in J} \left\{ A^j_n \cdot g_{nt} + B^j_n \cdot u_{nt} \right\}, \forall n \in N, \forall t \in T$$ \hspace{1cm} (5)$$

$$\sum_{t=1}^{t_n^u} (1 - u_{nt}) = 0, \forall n \in N$$ \hspace{1cm} (6)$$

$$\sum_{t=\tau}^{t_n^u-1} u_{nt} \geq T^u_n \cdot (u_{nt} - u_{nt(t-1)}), \forall n \in N, t = I^u_n + 1, ..., |T| - T^u_n + 1$$ \hspace{1cm} (7)$$

$$\sum_{t=\tau}^{t_n^d-1} \left( u_{nt} - (u_{nt} - u_{nt(t-1)}) \right) \geq 0, \forall n \in N, t = I^d_n + 1, ..., |T| - T^d_n + 1$$ \hspace{1cm} (10)$$

$$\sum_{n \in N} g^N_{nt} + \sum_{k \in K} \sum_{m \in M} q^N_{kt} = \sum_{m \in M} D_{mt}, \forall t \in T$$ \hspace{1cm} (12)$$

$$\sum_{m \in M} K_{bm} \left( \sum_{n \in N_m} g^N_{nt} + \sum_{k \in K_m} q^N_{kt} - D_{mt} \right) \leq F_b,$$ \hspace{1cm} (13)$$

$$\sum_{m \in M} K_{bm} \left( \sum_{n \in N_m} g^N_{nt} + \sum_{k \in K_m} q^N_{kt} - D_{mt} \right) \leq F_b,$$ \hspace{1cm} (13)$$

$$g^N_{nt} + r^U_{nt} \leq G^\text{max}_{nt} u_{nt}, \forall n \in N, \forall t \in T$$ \hspace{1cm} (15)$$

$$r^U_{nt} \geq 0,$$ \hspace{1cm} (16)$$

$$g^N_{nt} - r^D_{nt} \geq G^\text{min}_{nt}, \forall n \in N, \forall t \in T$$ \hspace{1cm} (17)$$

$$r^D_{nt} \geq 0,$$ \hspace{1cm} (18)$$

$$0 \leq q^N_{kt} \leq W^-_{kt}, \forall k \in K, \forall t \in T$$ \hspace{1cm} (19)$$

$$g^N_{nt(t-1)} - g^N_{nt} \leq RD_n \cdot u_{nt} + G^\text{max}_{nt} (1 - u_{nt}), \forall n \in N, \forall t \in T$$ \hspace{1cm} (20)$$

$$g^N_{nt} - g^N_{nt(t-1)} \leq RU_n \cdot u_{nt(t-1)} + G^\text{max}_{nt} (1 - u_{nt(t-1)}), \forall n \in N, \forall t \in T$$ \hspace{1cm} (21)$$
\[ \sum_{n \in N} g^Z_{nt}(z) + \sum_{k \in K} q^Z_{kt}(z) = \sum_{m \in M} D_{mt}, \quad \forall t \in T, \forall z \in Z \]  
(22)

\[ \sum_{m \in M} K_{bm} \left( \sum_{n \in N_m} g^Z_{nt}(z) + \sum_{k \in K_m} q^K_{kt}(z) - D_{mt} \right) \leq F_b, \quad \forall b \in B, \forall t \in T, \forall z \in Z \]  
(23)

\[ - \sum_{m \in M} K_{bm} \left( \sum_{n \in N_m} g^Z_{nt}(z) + \sum_{k \in K_m} q^K_{kt}(z) - D_{mt} \right) \leq F_b, \quad \forall b \in B, \forall t \in T, \forall z \in Z \]  
(24)

\[ g^Z_{nt}(z) \leq g^N_{nt} + r^U_{nt}, \quad \forall n \in N, \forall t \in T, \forall z \in Z \]  
(25)

\[ g^Z_{nt}(z) \geq g^N_{nt} - r^D_{nt}, \quad \forall n \in N, \forall t \in T, \forall z \in Z \]  
(26)

\[ 0 \leq q^Z_{kt}(z) \leq u_{kt}(z), \quad \forall k \in K, \forall t \in T, \forall z \in Z \]  
(27)

\[ g^Z_{nt}(z) - g^Z_{nt-1}(z) \leq RU_n \cdot u_{nt} + C_{n}\max \left(1 - u_{n(t-1)}\right), \quad \forall n \in N, \forall t \in T, \forall z \in Z \]  
(28)

\[ g^Z_{nt}(z) - g^Z_{nt-1}(z) \leq RU_n \cdot u_{nt} + C_{n}\max \left(1 - u_{n(t-1)}\right), \quad \forall n \in N, \forall t \in T, \forall z \in Z \]  
(29)

The objective function (3) is composed of the start-up cost, the power generation cost, and the cost of up and down reserve offered by generators. The start-up cost \( S^G_{nt} \) and generation cost \( C^G_{n} \) are defined as (4) and (5), respectively.

The minimum up and minimum down time constraints are expressed by (6)-(8), and (9)-(11), respectively. Constraints (12) represent the power balance between total generation and demand, and the transmission limitation is imposed by constraints (13)-(14). Expressions (15)-(19) are used to enforce generation and reserve capacity constraints in the nominal case, and the ramp-rate limitations are represented by constraints (20)-(21).

Constraints (22)-(29) ensure that there is at least one feasible redispach decision, under all realizations of uncertain wind power generation over the uncertainty set \( Z \). Similar to the dispatch decisions in nominal cases, the redispach decisions, in terms of thermal generation \( g^Z_{nt}(z) \) and wind power generation \( q^Z_{nt}(z) \), are subject to power balance constraint (22), transmission capacity constraints (23)-(24), generation capacity constraints (25)-(27), as well as the ramp-rate limitation constraints (28)-(29).

It should be noted that the UC decisions, as well as the energy and reserve dispatch in the nominal case are made before the observation of uncertain wind power generation, or in a here-and-now manner. The redispach decisions should be determined after the realization of uncertain wind power output, so they are wait-and-see decisions, or recourse decisions. It has been shown in [10], [26] that searching for a here-and-now decision that guarantees the existence of feasible recourse decisions for all realization over an uncertainty set is NP-hard. Thereby in the next subsection, these redispach decisions will be approximated by linear decision rules, in order to attain computational tractability.

### C. Linear Decision Rule Approximation for the Redispach Decisions under Uncertainty

Suppose that the uncertainty set \( Z \) is expressed as the following linear matrix form.

\[ Z = \{ z : H z \leq h, z \geq 0 \} \]  
(30)

Let \( x \) denote the unit status and the dispatch decisions under nominal scenarios, and \( y(z) \) be the redispach decisions under wind power realization \( z \), the UC formulation discussed above can be presented in a compact matrix form shown by (31)-(33).

\[ \min c^T x \]  
(31)

\[ s.t. \ Ax \leq b \]  
(32)

\[ U x + V y(z) \leq d(z), \forall z \in Z \]  
(33)

The detailed formulation given in section III.B. suggests that this is a fixed recourse problem [2], because only the right-hand-side vector \( d(z) \) is affected by the uncertain wind power output \( w_{kt}(z) \), as shown in constraints (27). It is further assumed that the right-hand-side vector \( d(z) \) in (33) can be expressed as linear affine function (34) of the uncertainty factors \( z \).

\[ d(z) = \hat{d} + \sum_{s \in S} \hat{d}^s z_s \]  
(34)

where \( S \) is the set of all uncertain variables.

This problem is generally intractable due to constraints (33) that involves recourse decisions \( y(z) \) under unresolved uncertainty. However, an upper bound of it can be computed tractably by restricting the recourse decision \( y(z) \) to be affinely dependent on the uncertain factors \( z \). This approximation, or so called linear decision rule [23], [10], can be expressed in the mathematical form shown by (35).

\[ y(z) = y^0 + \sum_{s \in S} y^s z_s \]  
(35)

Note that the constraints (33) hold under all realizations \( z \) over the uncertainty set \( Z \) if every constraint is satisfied under the worst-case realization over \( Z \), so expression (33) can be rewritten as constraint (36). More details of the derivation of this equivalent form can be found in reference [10].

\[ \left[ U x + \max_{z \in \bar{Z}} \{ V y(z) - d(z) \} \right] \leq 0, \forall l \in L \]  
(36)

The left-hand-side expression of constraint (36) can be rewritten as follows.

\[ \begin{align*}
U x + \max_{z \in \bar{Z}} \{ V y(z) - d(z) \} &= U^T x + \max_{z \in \bar{Z}} \left\{ V^T y^0 + \sum_{s \in S} V^T y^s z_s - \hat{d}_l - \sum_{s \in S} \hat{d}^s z_s \right\} \\
&= U^T x + V^T y^0 - \hat{d}_l + \sum_{s \in S} \left\{ V^T y^s - \hat{d}^s \right\} z_s \\
&= U^T x + V^T y^0 - \hat{d}_l + \min_{\lambda \in D_l} \{ \lambda^T h \}, \forall l \in L
\end{align*} \]  
(37)
where $U_i^T$ and $V_i^T$ are the $i$th row of the matrix $U$ and $V$, respectively in (33), and $\lambda_i$ is the dual solution associated with the maximization problem in (37), and $\mathcal{D}_i$ is the feasibility set of the dual variables, derived as the linear constraints below.

$$\mathcal{D}_i = \left\{ \lambda : \lambda^T H_s \geq V_i^T y^s - \tilde{d}_i^s, \forall s \in \mathcal{S} \right\}$$

(38)

where the vector $H_s$ is the $s$th column of the matrix $H$ in the uncertainty set $\mathcal{Z}$. According to weak duality [27], the objective value of the dual problem in (37) is always no lower than the objective value of the primal maximization problem, so the inequality (36) must hold if the following constraints are satisfied.

$$U_i^T x + V_i^T y^s - \tilde{d}_i \leq 0, \quad \forall l \in \mathcal{L}$$

(39)

$$\lambda_i \geq 0, \quad \forall l \in \mathcal{L}$$

(40)

The robust unit commitment is thus equivalent to the following mixed-integer linear programming (MILP) problem.

$$\min c^T x$$

s.t. \quad Ax \leq b$$

(42)

$$U_i^T x + V_i^T y^s - \tilde{d}_i \leq 0, \quad \forall l \in \mathcal{L}$$

(39)

$$\lambda_i \geq 0, \quad \forall l \in \mathcal{L}$$

(41)

$$\lambda_i H_s \geq V_i^T y^s - \tilde{d}_i^s, \quad \forall s \in \mathcal{S}, \forall l \in \mathcal{L}$$

(40)

It can be seen from the formulation above, especially constraints (45), that the dimension of the robust optimization problem greatly relies on the total number of uncertain variables, or the size of the set $\mathcal{S}$. In the implementation of linear decision rule, consequently, it is a common practice to assume that some recourse decisions only depend on a subset of all uncertain variables [28], [29]. Let $\mathcal{S}_i$ denote the subset of uncertain variables that the $i$th recourse decision depends on, and $\mathcal{I}$ be the set of all recourse decisions, the reduced decision rule is defined as follows.

$$y_i(z) = y_i^0 + \sum_{z \in S} y_i^z z_s, \quad \forall i \in \mathcal{I}$$

(47)

$$y_i^s = 0, \quad \forall s \in \mathcal{S}/\mathcal{S}_i, \forall i \in \mathcal{I}$$

(48)

Because the decision variable $y_i^z$ is always zero for uncertain factors excluded from the dependent subset $\mathcal{S}_i$, it can be removed from the formulation, thus both the number of variables and the number of constraints are reduced.

In the context of power systems, this reduced decision rule should also be applicable, because the redispatch decisions are unlikely to be affected by some of the uncertain variables. For example, the realization of wind power generation at the first hour may have almost no influence on the redispatch decisions made at the last hour of the operation horizon, so there is no need to include such weakly related uncertain variables into the decision rule. As a result, in this paper, redispatch decisions are assumed to be dependent only on the uncertainty variables in the corresponding time step.

### IV. Case Studies

Case studies are presented in this section to demonstrate the effectiveness of the proposed uncertainty model for wind power generation, and to show the computational efficiency of this robust UC formulation.

All numerical experiments are conducted based on one area of the IEEE Reliability Test System-1996 [24], which includes 24 buses, 32 generators, as well as three wind farms, located at bus 3, bus 12, and bus 18. The expected output of uncertain wind power generation is shown in Table I, and the maximum and minimum values of wind power generation at each bus are given in Table II.

It is assumed that the cost of No.6 oil is 8.4$/MMBTU$, and that of the No. 2 oil is 15.17$/MMBTU$. The price of coal and nuclear are 1.78$/MMBTU$ and 0.6$/MMBTU$, respectively, and the cost of hydro and wind energy is assumed to be negligible. The fuel cost function of each unit is approximated by a three-segment linear piecewise form. The reserve price offered by each generating unit is assumed to be 25% of the marginal generation cost of the third segment.

An important feature of robust optimization is that it models the impact of system uncertainties by a predefined uncertainty set, instead of using the detailed information of the exact probability distribution. The level of conservatism of UC decisions can be easily adjusted by the parameters of the uncertainty set. Case studies in this subsection are used to illustrate how the uncertainty budget $\Gamma$ affects the performance of generated UC decisions under uncertainty. Because there are totally three wind farms installed, in following numerical experiments, the uncertainty budget $\Gamma$ is changed from 0.3 to 3, while the other system parameters remain the same.

### TABLE I

**Expected Wind Power Generation (MW)**

<table>
<thead>
<tr>
<th>Hour</th>
<th>Bus 3</th>
<th>Bus 12</th>
<th>Bus 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>88.00</td>
<td>132.00</td>
<td>77.00</td>
</tr>
<tr>
<td>2</td>
<td>81.60</td>
<td>122.40</td>
<td>71.40</td>
</tr>
<tr>
<td>3</td>
<td>54.40</td>
<td>81.60</td>
<td>47.60</td>
</tr>
<tr>
<td>4</td>
<td>102.00</td>
<td>122.40</td>
<td>67.20</td>
</tr>
<tr>
<td>5</td>
<td>91.80</td>
<td>81.60</td>
<td>61.20</td>
</tr>
<tr>
<td>6</td>
<td>110.00</td>
<td>122.40</td>
<td>68.00</td>
</tr>
</tbody>
</table>

### TABLE II

**Maximum and Minimum Output of Uncertain Wind Power Generation (MW)**

<table>
<thead>
<tr>
<th>Bus 3</th>
<th>Bus 12</th>
<th>Bus 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Output</td>
<td>216.00</td>
<td>303.20</td>
</tr>
<tr>
<td>Min Output</td>
<td>44.00</td>
<td>40.80</td>
</tr>
</tbody>
</table>
The objective value of the robust UC problem under various values of uncertainty budget $\Gamma$ is illustrated by Fig. 1. It shows that as $\Gamma$ increases, the total cost rises steadily. This is because higher value of the parameter $\Gamma$ includes more scenarios of wind power output into the uncertainty set, so more generators are turned on, and more reserve is assigned to better protect power systems against higher level of uncertainties. This point is also supported by Fig. 2, which displays the results in terms of the number of online units, and Fig. 3, which shows the total reserve under various values of $\Gamma$.

The results above suggest that the UC and dispatch decisions yielded from the proposed robust optimization formulation greatly depend on the selection of the uncertainty set. Different levels of protection can be achieved by adjusting the uncertainty budget parameter $\Gamma$. Defining proper uncertainty sets based on the historical data and the preference over risk is therefore very important in the implementation of robust optimization approaches. More details of these techniques can be found in reference [8], [30].

All case studies are solved by CPLEX 12.51 on a Dell Latitude E6420 laptop, with a 2.20GHz Intel Core i7 CPU and 4GB RAM. The relative duality gap tolerance of the MILP problem is set to be $10^{-3}$. The computational experience of this robust optimization approach is presented in Table III. The average solution time of all ten cases is 81.5 seconds, and the longest solution time is no more than three minutes.

These experiments show that the proposed formulation can be solved quite efficiently, and it has also been proved in previous literatures [10], [26] that by applying the linear decision rule, the adjustable robust counterpart is computationally tractable, given that the problem is fixed recourse. This is an appealing feature for the operation of large power systems with multiple sources of uncertain renewable energy sources.

The tractability of the robust model can be illustrated by comparing its dimension with the stochastic programming approaches. The stochastic programming model uses a number of discrete scenarios to represent the distribution of uncertain system parameters, and this number of scenarios could be extremely large due to the multi-period structure of the problem. For example, if three scenarios are used to represent the uncertainty of one wind power generation source at each time step, the total number of scenarios over the 24-hour horizon can be as large as $3^{24}$. It is almost impossible to solve such a large-scale problem if multiple wind power generation sources are considered. The dimensions of the robust UC problem of the IEEE test case and the stochastic programming model considering different numbers of scenarios are shown in Table IV. It can be seen that the stochastic programming problems have much more constraints even when the number of scenarios is around 500, and the number of constraints exceeds the robust model when the number of scenarios is larger than 2500. This comparison suggests that the robust optimization formulation has smaller dimensions than the
TABLE IV
PROBLEM DIMENSION OF THE ROBUST UC AND STOCHASTIC PROGRAMMING FORMULATION

<table>
<thead>
<tr>
<th>Scenario No.</th>
<th>Variable No.</th>
<th>Constraint No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>R.O.</td>
<td>-</td>
<td>1875265</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1875265</td>
</tr>
<tr>
<td>1000</td>
<td>5264680</td>
<td>1065909</td>
</tr>
<tr>
<td>1500</td>
<td>12664680</td>
<td>1065909</td>
</tr>
<tr>
<td>2000</td>
<td>1684680</td>
<td>2524680</td>
</tr>
<tr>
<td>2500</td>
<td>2524680</td>
<td>1500</td>
</tr>
<tr>
<td>3000</td>
<td>2524680</td>
<td>2524680</td>
</tr>
</tbody>
</table>

stochastic programming model, even when only of small fraction of scenarios are sampled for the latter.

V. CONCLUSION

In this paper, we proposed a robust optimization formulation to address UC problems under uncertain wind power generation. This UC model minimizes the total cost of generation and reserve, while protecting power systems against all possible realization of wind power generation over a predefined uncertainty set. The uncertainty set can be constructed without knowing the exact probability distribution of uncertain wind output, so it might be more practical that the scenario-based stochastic programming or chance-constrained programming approaches. Besides, we have applied the linear decision rule techniques to approximate the wait-and-see redispetch decisions, so this UC problem remains computationally tractable. Our numerical experiments on the IEEE-RTS show that the level of risk-aversion can be well controlled by assigning proper parameters for the uncertainty set, and the computational experience demonstrate that this UC formulation can be solved in a timely manner.

This paper mainly focuses on developing a general robust optimization formulation for solving UC problems in the presence of wind power uncertainties. It is suggested by our case studies that the performance of this robust optimization framework greatly relies on the selection of uncertainty sets, so in our future research, we would like to explore the methods for designing appropriate uncertainty sets for wind power, as well as the other renewable energy sources. It would also be interesting to apply refined decision rule techniques, such as the deflected liner decision rule [28] to improve the performance of the robust UC model.

REFERENCE