An Affine Arithmetic Approach for Microgrid Dispatch with Variable Generation and Load

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Abstract—A self-validated computing (SVC) method, based on affine arithmetic (AA) is proposed in this paper to solve the optimal power flow (OPF) problem for microgrids with renewable sources of variable generation. In the AA-based OPF formulation, all the state and control variables are presented in affine form, to represent the variable load, and wind and solar generation. Hence, the OPF model becomes an interval-based model with upper and lower bounds to represent the uncertain variables. To check the accuracy of the AA-based method, the resulted intervals are compared against those obtained from Monte-Carlo Simulation (MCS), in a 13-bus microgrid test system. The obtained real power generation intervals for thermal generators are used to determine the reserves required in dispatchable generators in the short-term to properly supply for the variability of load and intermittent renewable generation sources.

Keywords—Optimal power flow, microgrids, generation uncertainty, load uncertainty, affine arithmetic, interval analysis.

I. INTRODUCTION

Improvements in technology and economics of distributed energy resources (DERs) has enhanced the popularity of microgrids in recent years, as these are capable of providing energy to remote areas and to consumers with sensitive loads, efficiently and economically. Microgrids are defined as clusters of coordinated distributed generations (DGs), such as energy storage system (ESS), wind turbines, photovoltaics (PVs), and combined heat and power (CHP), which can operate either in grid-connected or isolated mode in order to reduce the burden on the grid operator [1]. Microgrids have the potential to significantly reduce carbon emissions with the proper integration of renewable sources; improve power quality and reliability, as generation is close to the load; and increase overall energy efficiency through CHPs [2].

In order to reliably operate microgrids, short-term power system models such as optimal power flow (OPF) problems should be revised to consider the variability of wind speed and solar radiation, which affect wind turbine and PV power outputs.

Monte-Carlo Simulation (MCS) techniques have been widely used in the literature to study the uncertainties associated with some DG sources, and their corresponding impact on the reliability of the system. However, these methods require long execution times and an appropriate estimation of the probability distribution function (pdfs) associated with the corresponding uncertainties, which would be unrealistic or difficult to determine [3].

Stochastic methods, suggested to study variability of DGs in OPF problems, could also be applied to microgrids. One of the first attempts to solve the probabilistic OPF (P-OPF) problem is reported in [4], where the multivariate Gram-Charlier method is employed to model the pdf of uncertain variables. In [5], the error between the forecast and the actual demand is assumed to have a Gaussian probability distribution. The model is solved using MCS, and the mean and variance of the active power generation and system losses are calculated so that the dispatcher is able to allocate enough spinning reserve capacity for a given time interval. A sensitivity analysis based technique is proposed in [6] wherein operating constraint violations and their probabilities are determined for the whole planning horizon, followed by an iterative approach to adjust the control variables while satisfying their limits. In order to specifically consider the variations of wind speed, a method is developed in [7] to find the factors for over-estimation and under-estimation of wind power generation, considering a Weibull distribution for the wind speed variable and the pdf is shown to be significantly dependent on good approximations of the values of its parameters.

Relatively few approaches have been presented in the literature to approximate the required spinning reserve capacity during the shortage of renewable generation in microgrids. In [8], Expected Energy Not Served (EENS) and Loss of Load Probability (LOLP) indices are used as constraints in a market clearing problem without considering the wind power pdf, which is later considered in [9]. In [10] and [11], the proposed model in [9] is enhanced by aggregating the pdf of wind and load forecast into a unit commitment (UC) problem.

In order to overcome the difficulties associated with pdf approximations and improve the execution time of probabilistic approaches, interval-based methods have been suggested in the literature. An interval arithmetic (IA) method is used in [12] to determine strict bounds to the solution of the

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power flow problem for uncertain system parameters. The interval linear power flow equations are solved herein using iterative methods or employing explicit inverse of matrices to obtain the hull of the solution set. The IA method however lacks accuracy because of expansive intervals, with increasing sources of uncertainties and system size. To demonstrate some of the limitations of the IA approaches, self-validated computation (SVC) techniques based on Affine Arithmetic (AA) have been proposed in [13], [14] and [15] for power flow analysis, where uncertain variables are modeled in affine form. The AA-based method is shown to provide better bounds than IA, since it considers the correlation between the input variables.

The present paper proposes a novel application of AA techniques to the OPF problem in the context of microgrids. The objective is to model uncertainties associated with demand response and intermittent renewable generation without the use of pdfs, and subsequently determine thermal generation reserves required by the microgrid. The proposed method is tested and validated using a 13-bus microgrid test system, demonstrating the efficiency and accuracy of the method, using MCS as the benchmark for comparison purposes.

The rest of the paper is organized as follows: Section II presents a brief background on the AA method, and Section III proposes an AA-based OPF solution methodology. In Section IV, the proposed method is tested and validated on 13-bus microgrid. Finally, in Section V, the main conclusions and contributions of the paper are highlighted.

II. BACKGROUND TO AFFINE ARITHMETIC

Probabilistic methods consider the uncertainties associated with input variables and hence produce the expected values for outputs. Most of the methods rely heavily on statistical information of the uncertain variables, and require a significant execution time to generate random scenarios. Furthermore, most uncertainty analysis techniques, such as the MCS method, try to capture external uncertainties and neglect the impact of internal errors caused by approximation and truncation. To address these issues, SVC or Automatic Result Verification approaches such as the IA and AA methods have been proposed, which do not need the pdf of the input variables and also inherently keep track of the internal errors [16].

The IA method is the simplest self-validated range analysis technique, providing conservative bounds for interval values. However, one of the main disadvantages of IA is error explosion, resulting in very conservative final bounds, which are too wide to be useful. The main reason for the wider bounds in IA calculations is the independency problem, in which the correlation between IA values is neglected. The AA method, on the other hand is a range analysis technique that not only handles external uncertainties such as forecast errors, but also internal errors such as arithmetic roundoff, series truncation and function approximation [16]. Although the AA method is computationally more expensive than the IA method, it provides narrower intervals, thus justifying the extra cost, as it considers the correlation between the input data.

An affine representation $\tilde{x}$ of a value $x$ (e.g., variable or parameter) is represented in the following form:

$$\tilde{x} = x_\circ + x_1 \varepsilon_1 + x_2 \varepsilon_2 + \cdots + x_n \varepsilon_n$$  \hspace{1cm} (1)

Each noise variable $\varepsilon_i$, which lies between -1 and 1, represents an independent source of uncertainty, and each coefficient $x_i$ models the magnitude of that uncertainty. Each affine form can be converted to an interval form by adding or subtracting the summation of the absolute values of all noise magnitudes to or from the central value $x_\circ$ in order to find the upper bound $\bar{x}$ and lower bound $\underline{x}$, as follows:

$$[\bar{x}, \underline{x}] = [x_\circ - \sum_{i} |x_i|, x_\circ + \sum_{i} |x_i|]$$  \hspace{1cm} (2)

where $\sum_{i} |x_i|$ is called the total deviation of the affine form $\tilde{x}$.

A. Affine Operations [16]

Each affine operation $f(\cdot)$ computes an affine form $\tilde{y}$, which is consistent with the affine input values as follows. Thus, consider $\tilde{x}$ and $\tilde{y}$:

$$\tilde{x} = x_\circ + x_1 \varepsilon_1 + x_2 \varepsilon_2 + \cdots + x_n \varepsilon_n = x_\circ + \sum_{i=1}^{n} x_i \varepsilon_i$$  \hspace{1cm} (3)

$$\tilde{y} = y_\circ + y_1 \varepsilon_1 + y_2 \varepsilon_2 + \cdots + y_n \varepsilon_n = y_\circ + \sum_{i=1}^{n} y_i \varepsilon_i$$  \hspace{1cm} (4)

Then, the following elementary affine operations can be defined:

$$\tilde{z}_1 = \tilde{x} \pm \tilde{y} = (x_\circ \pm y_\circ) + (x_1 \pm y_1) \varepsilon_1 + \cdots + (x_n \pm y_n) \varepsilon_n$$  \hspace{1cm} (5)

$$\tilde{z}_2 = \varphi \tilde{x} = (\varphi x_\circ) + (\varphi x_1) \varepsilon_1 + \cdots + (\varphi x_n) \varepsilon_n$$  \hspace{1cm} (6)

$$\tilde{z}_3 = \tilde{x} \varphi \tilde{y} = (x_\circ \pm \varphi) + x_1 \varepsilon_1 + x_2 \varepsilon_2 + \cdots + x_n \varepsilon_n$$  \hspace{1cm} (7)

where $\varphi$ is a constant.

B. Non-Affine Operations [16]

Non-affine operations require an affine approximation and an extra term, called approximation error, to represent internal errors. For instance, a non-affine multiplication operation between the two affine values $\tilde{x}$ and $\tilde{y}$ can be written as:

$$\tilde{z} = \tilde{x} \tilde{y} = x_\circ y_\circ + \sum_{i=1}^{n} (x_\circ y_i + y_\circ x_i) \varepsilon_i + z_k \varepsilon_k$$  \hspace{1cm} (8)
The result of this non-affine function is an affine value containing the information provided by \( \bar{x} \) and \( \bar{y} \), and the approximation error represented by \( z_k e_k \), where the lower bound of \( |z_k| \) can be obtained as follows:

\[
|z_k| \geq \left( \sum_{i=1}^{n} x_i e_i \right) \left( \sum_{i=1}^{n} y_i e_i \right) \tag{9}
\]

The simplest and most conservative affine approximation (e.g., trivial affine approximation) can be calculated as follows:

\[
z_k = \sum_{i=1}^{n} |x_i| \sum_{i=1}^{n} |y_i| \tag{10}
\]

The affine approximation (10) is computationally efficient, but not the most accurate, because the error in this case is at most four times the error reported by the most accurate method, i.e., the Chebyshev approximation. However, it is used here given its simplicity. Furthermore, to reduce the number of noise variables, \( e_k \) is assumed to be 1 throughout this paper, i.e.,

\[
\bar{x} \bar{y} = x_0 y_0 + \sum_{i=1}^{n} (x_0 y_i + y_0 x_i) e_i + \sum_{i=1}^{n} |x_i| \sum_{i=1}^{n} |y_i| \tag{11}
\]

Although this would yield more conservative results, the contraction method described in [17] is used in this paper to minimize the noise variables by reducing the AA-form intervals, to provide more accurate bounds, as demonstrated by the results in Section IV.

III. PROPOSED AA-BASED OPF

In order to consider the uncertainties in microgrids with renewable sources, the AA method is used to solve a cost minimizing OPF problem. The following uncertain variables are modelled in affine forms, consisting of a center value and noise magnitudes as in (1): real and reactive power generation \( \bar{P}_i^G \) and \( \bar{Q}_i^G \); real and reactive power demand \( P_i^0 \) and \( Q_i^0 \); bus voltage magnitude \( \bar{V}_i \); real and imaginary components of bus voltages \( \bar{e}_i \) and \( \bar{f}_i \); real and imaginary components of bus currents \( \bar{I}_{ri} \) and \( \bar{I}_{im} \); and line current magnitudes \( I_{ij} \).

A. AA-based OPF Formulation

The following equations correspond to the rectangular form of the AA-based OPF problem:

\[
\min \ F(\bar{P}^G) = \sum_{i \in \mathcal{N}} \alpha_i \bar{P}_i^{G2} + \beta_i \bar{P}_i^G + c_i \tag{12}
\]

s.t.: \( \Delta \bar{P}_i \left( \bar{e}_i, \bar{f}_i, \bar{I}_{ri}, \bar{I}_{im}, \bar{P}_i^G, \bar{Q}_i^G \right) = 0 \quad \forall i \in \mathcal{N} \tag{13} \]

\( \Delta \bar{Q}_i \left( \bar{e}_i, \bar{f}_i, \bar{I}_{ri}, \bar{I}_{im}, \bar{P}_i^G, \bar{Q}_i^G \right) = 0 \quad \forall i \in \mathcal{N} \tag{14} \]

\( |\bar{P}_i| \leq \bar{P}_i^G \leq P_i^\max \quad \forall i \in \mathcal{N} \tag{15} \]

\( Q_i^\min \leq \bar{Q}_i^G \leq Q_i^\max \quad \forall i \in \mathcal{N} \tag{16} \]

\( I_{ij}^\min \leq \bar{I}_{ij} \leq I_{ij}^\max \quad \forall ij \in \mathcal{L} \tag{17} \]

\( V_i^\min \leq |\bar{V}_i| \leq V_i^\max \quad \forall i \in \mathcal{N} \tag{19} \]

where \( \mathcal{N} \) is the set of all buses in the microgrid, \( \mathcal{NPG} \) is the set of all generator buses, and \( \mathcal{L} \) is the set of all lines. The functions \( \Delta \bar{P}_i(\cdot) \) and \( \Delta \bar{Q}_i(\cdot) \) are the affine real and reactive power mismatches, and \( P_i^\min, P_i^\max, Q_i^\min, Q_i^\max, I_{ij}^\min, I_{ij}^\max \), \( V_i^\min \) and \( V_i^\max \) are minimum and maximum limits for real power, reactive power, line currents, and bus voltage magnitudes, respectively, at bus \( i \).

The forecast output ranges of renewable generators are used to calculate the center values of affine variables. The center values are obtained by solving a deterministic OPF, in which the medians of the given intervals for upper and lower bounds of real and reactive power demand \( \bar{P}_i \) and \( \bar{Q}_i \), and \( \bar{P}_i^0 \) and \( \bar{Q}_i^0 \) are considered deterministic demands as follows:

\[
P_i^0 = \frac{\bar{P}_i^0 + P_i^0}{2} \quad \forall i \in \mathcal{N} \tag{20}
\]

\[
Q_i^0 = \frac{\bar{Q}_i^0 + Q_i^0}{2} \quad \forall i \in \mathcal{N} \tag{21}
\]

where \( \mathcal{N}D \) is the set of all buses with uncertain demand. In this paper, uncertain wind and solar generation sources are treated as interval negative loads with a constant power factor, and are hence represented using (20) and (21) in the deterministic OPF.

To obtain the noise magnitudes of real and imaginary components of bus voltages, a sensitivity analysis is carried out, where the generation and demand are perturbed by a small magnitude (e.g., \( \pm 1\% \)) at each node. A deterministic OPF is solved for each of these variations, thus solving as many OPFs as the number of uncertain inputs in the system, which corresponds to the number of demand-response loads and renewable generators being studied. Therefore, the noise magnitudes can be obtained as follows [13]:

\[
e_i^j = \left. \frac{\partial e_i^j}{\partial P_j^0} \right|_0 = \frac{e_i^N - e_i^0}{\Delta P_j^0} \quad \forall i, j \in \mathcal{N} \tag{22}
\]
where \( e_{i,j}^p \) and \( f_{i,j}^p \) are partial deviations of real and imaginary components of bus voltages due to changes in real power injection, and \( e_{i,j}^0 \) and \( f_{i,j}^0 \) are partial deviations of real and imaginary components of bus voltages at bus \( i \) due to changes in reactive power injection at bus \( j \). The parameters \( e_{i,j}^p \) and \( f_{i,j}^p \) are the new real and imaginary components of the bus voltages when the real and reactive power injections are perturbed; \( e_{i,j}^0 \) and \( f_{i,j}^0 \) are the initial values of real and imaginary components of bus voltages obtained from the deterministic model; and \( \Delta P_{ij}^p \) and \( \Delta Q_{ij}^p \) are the amount of perturbation in real and reactive power injections at bus \( j \).

The affine forms of the real and imaginary components of the bus voltage magnitude \( \tilde{e}_i \) and \( \tilde{f}_i \), which are linear functions of noise variables \( e_{ij}^p \) and \( e_{ij}^0 \) representing the uncertainties of active power and reactive power injections at bus \( j \), can then be presented as follows:

\[
\tilde{e}_i = e_{i,0} + \sum_{j \in N \setminus \{i\}} e_{i,j}^p e_{j,p} + \sum_{j \in N \setminus \{i\}} e_{i,j}^0 e_{j,0}^p \quad \forall i \in N \tag{26}
\]

\[
\tilde{f}_i = f_{i,0} + \sum_{j \in N \setminus \{i\}} f_{i,j}^p e_{j,p} + \sum_{j \in N \setminus \{i\}} f_{i,j}^0 e_{j,0}^p \quad \forall i \in N \tag{27}
\]

where \( e_{i,0} \) and \( f_{i,0} \) are the center values for real and imaginary components of bus voltages at bus \( i \), respectively, and \( e_{i,j}^p, f_{i,j}^p, e_{i,j}^0, \) and \( f_{i,j}^0 \) are defined in (22)-(25).

The affine forms of real and imaginary components of bus voltages, \( \tilde{e}_i \) and \( \tilde{f}_i \) can be used to calculate, from (11), the square of bus voltage magnitude (15). Thus, \( |P_i|^2 \) has the following form:

\[
|P_i|^2 = (\tilde{e}_i^2 + \tilde{f}_i^2) + 2 \sum_{j \in N \setminus \{i\}} (e_{i,0} e_{i,j}^p + f_{i,0} f_{i,j}^p) e_{j,p} + \sum_{j \in N \setminus \{i\}} (e_{i,0} e_{i,j}^0 + f_{i,0} f_{i,j}^0) e_{j,0}^p + \left( e_i^2 + f_i^2 \right) \quad \forall i \in N \tag{28}
\]

where \( e_i^2 \) and \( f_i^2 \) are truncation errors due to non-affine operations. Knowing the affine forms of the real and imaginary components of the bus voltage magnitude, the real and reactive power can be calculated using the following affine operations:

\[
\tilde{I}_i = \sum_{j \in N} (G_{ij} + jB_{ij})(\tilde{e}_j + j\tilde{f}_j) = \tilde{I}_r + j\tilde{I}_{im} \quad \forall i \in N \tag{29}
\]

where \( j = \sqrt{-1} \), and \( G_{ij} \) and \( B_{ij} \) are the real and imaginary components of the \( Y \)-bus matrix, respectively, and the linear affine forms of real and imaginary bus currents \( \tilde{I}_r \) and \( \tilde{I}_{im} \) have the following general forms after affine operations:

\[
\tilde{I}_r = I_{r,0} + \sum_{j \in N} I_{r,j}^p e_{j,p} + \sum_{j \in N} I_{r,j}^0 e_{j,0}^p \quad \forall i \in N \tag{30}
\]

\[
\tilde{I}_{im} = I_{im,0} + \sum_{j \in N} I_{im,j}^p e_{j,p} + \sum_{j \in N} I_{im,j}^0 e_{j,0}^p \quad \forall i \in N \tag{31}
\]

Here, \( I_{r,0} \) and \( I_{im,0} \) are the center values for real and imaginary components of current magnitudes; \( I_{r,j}^p \) and \( I_{r,j}^0 \) are partial deviations of the real component of current at bus \( i \) for deviation in real and reactive power injection at a bus \( j \), respectively; and \( I_{im,j}^p \) and \( I_{im,j}^0 \) are partial deviations of imaginary component of current at bus \( i \) for deviation in real and reactive power injection at bus \( j \), respectively. Note that the real and imaginary components of the current share the same sources of uncertainties, i.e., real and reactive power injections \( e_{j,p} \) and \( e_{j,0} \).

Using affine and non-affine operations and \( \tilde{e}_i, \tilde{f}_i, \tilde{I}_r, \) and \( \tilde{I}_{im} \), the real and reactive power mismatch \( \Delta P_i(\cdot) \) and \( \Delta Q_i(\cdot) \) in (13) and (14), respectively, can be calculated as follows:

\[
\Delta \tilde{P}_i(\tilde{e}_i, \tilde{f}_i, \tilde{I}_r, \tilde{I}_{im}) = \tilde{P}_i - \tilde{e}_i \tilde{I}_r - \tilde{f}_i \tilde{I}_{im} = 0 \quad \forall i \in N \tag{32}
\]

\[
\Delta \tilde{Q}_i(\tilde{e}_i, \tilde{f}_i, \tilde{I}_r, \tilde{I}_{im}) = \tilde{Q}_i - \tilde{e}_i \tilde{I}_r - \tilde{f}_i \tilde{I}_{im} = 0 \quad \forall i \in N \tag{33}
\]

where \( \tilde{P}_i \) and \( \tilde{Q}_i \) represent the affine real and reactive power injections, and have the following affine forms, with center value and associated partial deviations:

\[
\tilde{P}_i = P_{i,0} + \sum_{j \in N} P_{i,j}^p e_{j,p} + \sum_{j \in N} P_{i,j}^0 e_{j,0}^p + P_i^T \quad \forall i \in N \tag{34}
\]

\[
\tilde{Q}_i = Q_{i,0} + \sum_{j \in N} Q_{i,j}^p e_{j,p} + \sum_{j \in N} Q_{i,j}^0 e_{j,0}^p + Q_i^T \quad \forall i \in N \tag{35}
\]
where $P_0$ and $Q_0$ are the center values of affine real and reactive power injections; $P^p_{i,j}$ and $Q^p_{i,j}$ are the partial deviations of real and reactive power injections due to changes in real power injections at a bus $j$, respectively; $P^q_{i,j}$ and $Q^q_{i,j}$ are the partial deviations of real and reactive power injections due to changes in reactive power injections at bus $j$, respectively; and $P^r_i$ and $Q^r_i$ are real and reactive power injection truncation errors, based on (11). This formulation, as previously mentioned, is the most conservative but computationally efficient for calculating the magnitude of the internal errors $P^r_i$ and $Q^r_i$. The affine forms (34) and (35) for real and reactive powers can be represented in interval forms $[P_i, P_i]$ and $[Q_i, Q_i]$, where:

$$P_i = P_{i0} + radP_i(\epsilon_{p,i}, \epsilon_{q,i}) \quad \forall i \in N$$

$$Q_i = Q_{i0} + radQ_i(\epsilon_{p,i}, \epsilon_{q,i}) \quad \forall i \in N$$

Here, $radP_i(\epsilon_{p,i}, \epsilon_{q,i})$ and $radQ_i(\epsilon_{p,i}, \epsilon_{q,i})$ are the following functions of noise variables $\epsilon_{p,i}$ and $\epsilon_{q,i}$, with the most conservative values when they are equal to 1, and represent the total amount of deviation from the center value:

$$radP_i(\epsilon_{p,i}, \epsilon_{q,i}) = \sum_{j \in N} P^p_{i,j} \epsilon_{p,j} + \sum_{j \in N} P^q_{i,j} \epsilon_{q,j} + P^r_i$$

$$radQ_i(\epsilon_{p,i}, \epsilon_{q,i}) = \sum_{j \in N} Q^p_{i,j} \epsilon_{p,j} + \sum_{j \in N} Q^q_{i,j} \epsilon_{q,j} + Q^r_i$$

These intervals can be used to determine the operational range of generation for dispatchable generators (e.g., thermal, hydro), when there are sources of uncertainty such as renewable generation in the system.

### B. Contraction Method

The obtained intervals from (36)-(39) are conservative, since these are based on the maximum value of $\epsilon$. Thus, a contraction method, based on [17], is used here to find the minimum noise variables, while all the system operating constraints (e.g. voltage and generation limits) are respected. Hence, by replacing in the AA-based OPF model (12)-(19), all uncertain variables with their AA forms defined in (26)-(35), one obtains the following representation of the OPF model:

$$\min \tilde{F}(\epsilon_{p,i}, \epsilon_{q,i}) \quad (42)$$

s.t.: $\Delta \tilde{P}_i(\epsilon_{p,i}, \epsilon_{q,i}) = 0 \quad \forall i \in N \quad (43)$

$\Delta \tilde{Q}_i(\epsilon_{p,i}, \epsilon_{q,i}) = 0 \quad \forall i \in N \quad (44)$

$p^\text{min}_i \leq P_i(\epsilon_{p,i}, \epsilon_{q,i}) \leq p^\text{max}_i \quad \forall i \in \text{NPG} \quad (45)$

$q^\text{min}_i \leq Q_i(\epsilon_{p,i}, \epsilon_{q,i}) \leq q^\text{max}_i \quad \forall i \in \text{NPG} \quad (46)$

$$I^\text{min}_{ij} \leq \tilde{I}_{ij}(\epsilon_{p,i}, \epsilon_{q,i}) + \tilde{I}_{imij} \leq I^\text{max}_{ij} \quad \forall ij \in L \quad (47)$$

where the objective function $\tilde{F}(.)$ is the affine linear expansion of (12), as follows:

$$\tilde{F}(\epsilon_{p,i}, \epsilon_{q,i}) = \sum_{i \in N} a \left\{ \sum_{j \in N} \left( P_{i0}^2 + \sum_{j \in N} 2P_{i0} P^p_{i,j} \epsilon_{p,j} \right) + \sum_{j \in N} 2P_{i0} P^q_{i,j} \epsilon_{q,j} + P^r_i \right\} + \sum_{j \in N} b \left( \sum_{j \in N} P^p_{i,j} \epsilon_{p,j} + \sum_{j \in N} P^q_{i,j} \epsilon_{q,j} + P^r_i \right) + C_i$$

Note that in the above equation, due to an affine product, a new error magnitude $P^r_i$ is introduced. The line currents $\tilde{I}_{ij}$ in (47) are calculated as follows:

$$\tilde{I}_{ij} = G_{ij}(\tilde{e}_i - \tilde{e}_j) - B_{ij}(\tilde{f}_i - \tilde{f}_j) \quad \forall ij \in L \quad (50)$$

$$\tilde{I}_{imij} = G_{ij}(\tilde{e}_i - \tilde{e}_j) + B_{ij}(\tilde{f}_i - \tilde{f}_j) \quad \forall ij \in L \quad (51)$$

Observe that the OPF model (42)-(48) is a Linear Programming (LP) problem, since all affine forms are linear functions of the noise variables $\epsilon_{p,i}$ and $\epsilon_{q,i}$. This linear noise contraction model can be easily solved using available LP solvers, such as CPLEX [18].

Figure 1 depicts the procedure used to calculate the intervals for the affine variables and hence arrive at the solution intervals to the OPF with uncertainties.
demand intervals at each bus are the input of the model. These intervals consider the uncertainties associated with both demand and renewable generation by considering the latter as negative loads. Then affine variables $\tilde{e}_i$ and $\tilde{f}_i$ are constructed using both a deterministic OPF model to obtain center values, and a sensitivity analysis based on several perturbed OPFs to obtain the noise magnitudes. Affine operations are then applied on these affine variables in order to calculate other affine variables, such as $\tilde{P}_i$ and $\tilde{Q}_i$. To minimize the size of the intervals, a contraction method is used to minimize the noise magnitudes associated with each of these variables.

IV. CASE STUDY

The proposed AA-based OPF approach is applied to an isolated 13-bus microgrid test system, shown in Fig. 2 [19]. The intermittency effects of wind and solar generation is assumed to be balanced by thermal generation via continuous regulation, and thus the proposed AA-based method is used to determine the thermal generation reserve needed to reliably and optimally supply the demand. The studied microgrid system contains different renewable generation sources, such as PV and wind turbine, ESS (assumed here to be operating in discharging mode, for the considered dispatch interval), and diesel generator. The total installed capacity of the microgrid is 9,159 kW, from which 6,110 kW is the total capacity of the three diesel generators, installed at Bus 1, Bus 9 and Bus 13. The total capacity of storage units is 1,339 kW, and the total installed renewable generation capacity is 1,710 kW, i.e., 210 kW of PV capacity and 1,500 kW of wind capacity. The total system demand is considered to be 4,149 kW at the dispatch interval under consideration. It is assumed that the output levels of storage units, PVs, and wind turbines are 70%, 80%, and 50% of their maximum capacity, respectively. Furthermore, the renewable generators’ power outputs vary over a ±10% range of this center (forecast) value, and are treated as affine variables and appear to the system as negative loads. Note that, even though DR is not considered here, it could be modeled as affine variables in the same way and its interval of uncertainty can be obtained using the proposed approach.

The proposed model is simulated in the General Algebraic Modeling System (GAMS) [20] platform. The solvers used, i.e., COINPOPT for the nonlinear OPF solution, and CPLEX for the LP contraction problem, have their parameters set at their respective default, off-the-shelf settings, so as not to bias their “standard” performance. Major settings such as tolerance level or maximum number of iterations of the solver are by default the same for all solvers (e.g., feasibility tolerance is $10^{-6}$).

It is noted from the case solution that the expected values using the MCS method converge after 1400 iterations, assuming that the uncertain parameters have uniform distribution within the bounds defined for the assumed variable generation. The intervals obtained from the proposed AA-based approach are compared with those obtained from the MCS method, in order to check the accuracy of their
The intervals of operation of the diesel generators in the microgrid define the system reserves for reliable operation, in the presence of intermittent renewable generation. As shown in Table I, the maximum total thermal generation requirements, when renewable generation is below its center (forecast) value by 10%, is determined to be 2,557 kW using the MCS approach, while it is 2,715 kW with the AA-based approach. This results in a difference of only 6%, which depicts the accuracy of the proposed AA-based approach.

V. CONCLUSIONS

A novel AA-based model has been proposed to solve the OPF problem with intervals to represent system uncertainty such as variable wind and solar power generation, in microgrids. The efficiency and accuracy of the proposed AA-based OPF method, was tested on a 13-bus microgrid test system, and benchmarked against the MCS method. The AA-based approach was shown to efficiently yield adequate intervals which are slightly more conservative than MCS-based method. The intervals obtained from the proposed technique can be used to approximate the dispatchable resources reserves, needed to properly account for system uncertainties.

REFERENCES


1892.


