Novel Impedance Determination Method for Phase-to-Phase Loops

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Abstract—In this paper a novel impedance determination method for the phase-to-phase loops is described. This approach promises a better accuracy of the reactance calculation of the faulty loops. Thus, better selectivity and security in operation of the distance element applied for the protection of transmission and distribution lines is guaranteed. Moreover, with this new method the separation of the fault resistance from load condition is achieved. Due to this fact, special measures for load encroachment are not necessary. It contributes to simplification of distance protection settings.

Keywords—impedance determination; distance element; line protection; reactance method; transition resistance separation

I. INTRODUCTION

The basic idea of distance protection is to determine the impedance and compare it with the distance protection settings. The basic settings are applied as the so called protection zone, represented in the complex impedance plane. If the computed impedance is located in the set protection zone the distance element is picked up. The conventional distance element applied for the AC power system determines impedances from six possible loops. Hereby three phase-to-earth and three phase-to-phase loops are measured. So far only one technique is commonly used for calculating the phase-to-phase impedances. However, this method reveals various weaknesses. The results acquired by the commonly applied method are strongly impacted by network configuration as well as pre-fault and fault conditions in the power system. Both load flow and a strong remote-end infeed can contribute to significant deviation of the calculated impedance from expected value. Moreover under high load flow the so called apparent or ghost impedances from non-faulty loops can move into the set zone reach and cause unintended pickup of the distance element. In this paper a novel impedance calculation method with a reactance characteristic that is applied for the phase-to-phase loops is presented. This reactance method allows for the compensation of the load influence on the measured reactance as well as for the separation of the fault resistance. Due to this fact, the method offers a lot of advantages like direct reduction of over- and under-reach effects or the separation of the apparent impedance of non-faulty loops from the faulted loops. As a result, additional protection settings like load encroachment are not necessary anymore. The computed impedance with this method results in the fault reactance X and fault resistance Rf (electric arc resistance). The already existing reactance method as applied for single phase loops can be extended to include the proposed reactance element for the phase-to-phase loops. Consequently, a full scheme distance protection with reactance elements can be provided. It makes the distance settings significantly easier and better interpretable. The paper presents the derivation of the reactance method for the phase-to-phase loops. Detailed discussion and critical analysis of the reactance method in the phase-to-phase application supported with practically oriented scenarios are carried out.

II. STATE OF THE ART

The conventional distance protection algorithm when applied to the three phase power system measures the impedances with the six possible loops. Thus, the three phase-to-earth as well as the three phase-to-phase impedances are acquired in each measurement cycle. From a performance point of view the distance element utilizes simplified equations for impedance calculation. They are based on Kirchhoff’s laws applied for fundamental components as follows:

\[ U_{Ph-E} = Z_{Ph-E} \cdot I_{Ph} - Z_{Ph-E} \cdot k_E \cdot I_E, \]

\[ U_{Ph-Ph} = Z_{Ph-Ph} \cdot I_{Ph-Ph}, \]

\[ Z_{Ph-E} = \frac{U_{Ph-E} - k_E \cdot I_E}{I_{Ph}} \]

\[ Z_{Ph-Ph} = \frac{U_{Ph-Ph}}{I_{Ph-Ph}} \]

where \( Z_{Ph-E} \) is the measured complex impedance for a phase-to-earth loop and \( Z_{Ph-Ph} \) is the measured complex impedance for a phase-to-phase loop. The \( U_{Ph-E} \) is the phase-to-earth voltage phasor and \( U_{Ph-Ph} \) denotes the voltage phasor between two phases. The \( I_{Ph} \) is the current phasor in the considered phase \( Ph; I_E \) is the earth current phasor. \( I_{Ph-Ph} \) symbolizes the current difference between two phase currents. The so called residual compensation factor \( k_E \) reflects the ratio between an earth path represented by the earth path impedance \( Z_E \) and a phase conductor represented by the positive sequence impedance \( Z_l \).
The influence of fault resistance on the measured reactance with the phase to earth loop can be reduced by considering the \( k_e \) factor in a split form with the ratios \( k_e \) and \( k_k \) applied separately for the resistance and reactance 0. As a result, a modified formula for the calculation of the impedance is applied. The impedance result is given as positive sequence reactance and positive sequence resistance of the phase plus a fraction of the fault resistance. In consideration of a double side supplied network, the measurement of the impedance by means of the commonly used methods is only accurate if a metallic fault occurs (no fault resistance) and the considered loop is also the faulty one. If a fault with fault resistance (equivalent of electric arc) appears, the computed impedance result deviates from the real value and contributes to under- or over-reach of the distance element. This effect has negative impact on selectivity and security of the distance protection.

If one considers the system from Fig. 1, it can be observed that the equations (1)-(2) do not include the voltage drop across the fault resistance \( R_f \). The current \( I_f \) flowing through fault resistance is unknown with single side acquired quantities (principle of the distance element). As a result a deviation in calculated result for both reactance and resistance is observed.

The impedance result as reactance and resistance value is influenced by the following parameters:

- network homogeneity degree
- load flow
- fault resistance

The summarized impact of these parameters is graphically illustrated in Fig. 2. Dependent on the load flow direction and system homogeneity degree, a positive or negative reactance error is obtained. This error contributes to the overreach or under-reach effect of the distance element. The degree of the measured reactance error depends on the complex ratio of the two currents \( L_e \) and \( L_b \). These currents are mostly impacted by load flow so that clear tendency according to the error developing can predicted based on consideration load flow direction in most cases. Fault resistance works as an amplification factor. The approximated reactance error can be represented in following form:

\[
\Delta X = -R_f \left[ \frac{L_e + L_b}{L_e} \right] \sin(\angle (L_e + L_b, L_e))
\]

where \( L_e \) and \( L_b \) are the currents of both line sides. One can observe that not only the amplitude has influence on the reactance error but also the angle between currents from both ends of the line. If the fault resistance is zero then no measuring error for reactance \( X \) is expected. The same behavior is expected if the no loaded line is supplied from one side.

As mentioned this angle depends on the load flow and network homogeneity degree. Generally, the resistance in the loop cannot be accurately measured because it strongly depends on the short circuit power of the remote side source. However, this value is not required to reflect the fault location on the line. The reactance value is critical for this purpose.

Considering a simplified model that includes fault resistance but neglects the impedance of the return path, the correct equation for the impedance calculation can be written as follows:

\[
U_{ph} = Z_{fault} \cdot I_{ph} + R_f \cdot I_f,
\]

where \( Z_{fault} \) is the impedance between measuring point (relay) and short circuit location. \( I_f \) describes the current flowing through the fault resistance \( R_f \).

In order to reduce the influence of the voltage drop across the fault resistance, the so called compensation quantity with additional compensation angle must be introduced. As a result the equation can be transformed into the following form:

\[
U_{ph} \cdot L_{comp} e^{-j\delta_{comp}} = Z_{fault} \cdot L_{comp} e^{-j\delta_{comp}} + R_f \cdot L_{comp} e^{-j\delta_{comp}},
\]

where \( L_{comp} \) is the compensation quantity and \( \delta_{comp} \) the compensation shift angle. Since \( L_{comp} \) is not affected by load current and \( \delta_{comp} \) is the angle reflecting the degree of system non-homogeneity. The electric angle between fault current \( I_f \) and \( L_{comp} e^{j\delta_{comp}} \) interpreted as substitution for \( L_{ph} \) current is zero.

In consideration of only imaginary part of above equation, the influence of the fault resistance is finally eliminated as follows:

\[
X_{fault} = \frac{\sin \varphi \cdot \left[ \text{Im}[U_{ph} \cdot L_{comp} e^{-j\delta_{comp}}] \right]}{\text{Im}[e^{j\varphi} \cdot L_{ph} \cdot L_{comp} e^{-j\delta_{comp}}]},
\]

where \( X_{fault} \) is the fault reactance. The angle \( \varphi \) denotes the electric angle of the transmission or distribution line to be protected. Here the level of the compensation quantity is...
irrelevant. For the computation of the transition resistance \( R_f \) the following expression should be used:

\[
U_{ph'} \cdot L_{ph'} \cdot Z_{F,0} \cdot \delta_{Cmp,0} = Z_{F,0} \cdot L_{ph'} \cdot Z_{F,0} \cdot \delta_{Cmp,0},
\]

\( \delta_{Cmp,0} \) = necessary for full compensation. As mentioned this angle depends on the network parameters and can be computed for zero sequence as follows:

\[
\delta_{Cmp,0} = \arg \left( \frac {Z_{1,0} + Z_{g,0} + Z_{l,0}} {1 - m} \right),
\]

where \( Z_{l,0} \) is the zero sequence impedance of the source A, \( Z_{g,0} \) is the zero sequence impedance of the source B, \( Z_{l,0} \) is the zero sequence line impedance and \( m \) is fault position. \( \delta_{Cmp,0} \) is the compensation angle for the zero sequence component.

\( U \) is the zero sequence impedance of the source B, and \( \delta_{Cmp,0} \) is the zero sequence compensation angle for the zero sequence component.

In this case the imaginary part should be considered as well. As a result the transition resistance \( R_f \) can be expressed as follows:

\[
R_f = \frac {\text{Im}[U_{ph'} \cdot L_{ph'} \cdot \delta_{Cmp} \cdot I_{ph}]} {\text{Im}[L_f \cdot Z_{F,0} \cdot \delta_{Cmp} \cdot I_{ph}]}.
\]

It is apparent that the fault resistance value is strongly dependent on the assumed current \( I_f \) (Eq. 8). The computation of the impedance based on compensation method leads to extracting the reactance \( X_{Fault} \) and the fault resistance \( R_f \). For this reason this approach is called reactance method with separation of the fault resistance [4]-[5].

So far the reactance method was applied only for the phase-to-earth faults [2]-[5]. In this case the residual component of the loop should be considered. The equation can be transferred into the following form:

\[
X_{Fault} = \frac {\sin \phi \cdot \text{Im}[U_{ph'} \cdot L_{ph'} \cdot e^{-j\delta_{Cmp}}]} {\text{Im}[e^{j\phi} \cdot (L_{ph'} - L_{ph}) \cdot I_{ph} \cdot e^{j\delta_{Cmp}}]}.
\]

\( R_f = \frac {\text{Im}[U_{ph'} \cdot Z_{1,0} \cdot (L_{ph'} - L_{ph}) \cdot I_{ph}]} {\text{Im}[L_f \cdot Z_{l,0} \cdot (L_{ph'} - L_{ph}) \cdot I_{ph}]}.
\]

In three phase systems negative \( I_f \) or zero-sequence \( I_f \) currents are used as compensating values \( I_{Cmp} \) as they are decoupled from the load. These sequence currents result only from the passive network parameters and can be used uncomplicated as compensation quantities. It is illustrated in Fig. 3 based on the single phase-to-earth fault. The equivalent circuit in symmetrical components for this case consists of the serial connected circuits describing propagated current for each symmetrical component separately. The dashed line in the figure denotes the location of the relay. The dotted line denotes circulated currents that relay measures at installation point.

The positive sequence circuit includes the sources which are responsible for the generating current. In contrast to negative and zero sequence circuits in this circuit load flow through the line is reflected so that compensating with pure positive sequence is not possible. Since the negative and zero sequence currents are only available from one side of the line, the additional compensation angle \( \delta_{Cmp} \) is necessary for full compensation. As mentioned this angle depends on the network parameters and can be computed for zero sequence as follows:

\[
\delta_{Cmp,0} = \arg \left( \frac {Z_{l,0} + Z_{g,0} + Z_{l,0}} {1 - m} \right),
\]

where \( Z_{l,0} \) is the zero sequence impedance of the source A, \( Z_{g,0} \) is the zero sequence impedance of the source B, \( Z_{l,0} \) is the zero sequence line impedance and \( m \) is fault position. \( \delta_{Cmp,0} \) is the compensation angle for the zero sequence component.

III. REACTANCE METHOD FOR PHASE-TO-PHASE LOOPS

The basic idea of the load flow and network inhomogeneity degree compensation can be extended to multi-phase faults. This innovative approach is presented in this

![Fig. 3. Single phase-to-earth fault with fault resistance (representation in symmetrical components – double side supplied line) ![Fig. 4. Impedance comparison between conventional and reactance method](image)
section. The possible multi-phase faults for which the new method was developed are illustrated in Fig. 5.

At first a phase-phase fault with fault resistance is considered. The equation that describes the fault can be expressed in the following way:

\[
U_{F \text{-} Ph-Ph} = Z_{\text{fault}} \cdot I_{F \text{-} Ph-Ph} + R_F \cdot I_F, \tag{11}
\]

where \(U_{\text{F-Ph-Ph}}\) is the phase-to-phase voltage, \(I_{\text{F-Ph-Ph}}\) is the calculated current derived from the two phase currents and \(I_F\) is the current flowing in the fault resistance. The actual current \(I_F\) cannot be measured from any one side therefore the approximated value of this current is used here. For that the equivalent circuit in symmetrical components has to be considered (Fig. 6) [6]-[7]. For the phase-to-phase fault the circuit in symmetrical component consists of two parallel connected circuits reflecting propagated current for each component separately. The positive sequence includes sources so that this component cannot be used as compensating quantity.

Since the zero sequence component does not exist during the phase-phase fault without earth, the current \(I_F\) can be represented with pure negative sequence component:

\[
\begin{align*}
L_{\text{Ph-Ph}} &= L_{\text{F-Ph-Ph}} - L_{\text{F-Ph-Ph}}^2, \\
L_{\text{F-Ph-Ph}} &= (a^2 I_1 + a I_2), \\
L_{\text{F-Ph-Ph}}^2 &= (a I_1 + a^2 I_2), \\
I_1 &= -I_2, \\
L_{\text{Ph-Ph}} &= (a^2 I_2 + a I_1) - (a I_1 + a^2 I_2) = 2(a - a^2) \cdot I_2,
\end{align*}
\tag{12}
\]

where \(I_1\) and \(I_2\) is positive and negative sequence current as shown in Fig 6. The signs of both components are opposite. Also in this case the current \(I_F\) is completely decoupled from load flow conditions. The negative sequence current \(I_{\text{L-2}}\) acquired from left side is not sufficient to compensate the influence of the network non-homogeneity. Consequently, an additional compensation angle must be introduced here. This angle describes the homogeneity degree in negative sequence source impedance:

\[
\delta_{\text{Cmp},2} = \arg \left\{ \frac{Z_{\text{A,2}} + Z_{\text{B,2}} + Z_{\text{m,2}}}{(1 - m)Z_{\text{A,2}} + Z_{\text{B,2}}} \right\}, \tag{13}
\]

where \(Z_{\text{A,2}}\) is negative sequence impedance of the source A, \(Z_{\text{B,2}}\) is negative sequence impedance of the source B, \(Z_{\text{m,2}}\) is the negative sequence line impedance and \(m\) is the fault position. \(\delta_{\text{Cmp},2}\) is the compensation angle for the negative sequence current.

The compensation quantity is expressed in the following form:

\[
L_{\text{Cmp}(\text{Ph-Ph})} = 2 \cdot (a - a^2) \cdot L_{\text{L-2}} \cdot e^{j\delta_{\text{Cmp},2}}, \tag{14}
\]

Using the same procedure as for single phase-to-earth loops the exact reactance \(X_{\text{Fault}}\) and the approximated fault resistance \(R_F\) can be determined as follows (\(L_{\text{Cmp}(\text{Ph-Ph})}\) includes \(L_{\text{Cmp},2}\) and \(\delta_{\text{Cmp},2}\)):

\[
\begin{align*}
X_{\text{Fault}} &= \frac{\sin \phi \cdot \text{Im}(U_{\text{Ph-Ph}} \cdot L_{\text{Cmp}(\text{Ph-Ph})})}{\text{Im}(e^{j\phi} \cdot U_{\text{Ph-Ph}} \cdot L_{\text{Cmp}(\text{Ph-Ph})})}, \\
R_F &= \frac{\text{Im}(U_{\text{Ph-Ph}} \cdot Z_{\text{F-Ph-Ph}})}{\text{Im}(2 \cdot (a - a^2) \cdot L_{\text{L-2}} \cdot Z_{\text{m,2}} \cdot L_{\text{Ph-Ph}})}.
\end{align*}
\tag{15}
\]

In case of the double phase-to-earth faults the symmetrical representation of the fault is more complex. In this situation the load coupled component in form of the positive sequence must be replaced by the load decoupled components in form of the negative and zero sequence current (Fig. 7):

\[
L_{\text{L}} = L_{\text{L-2}} - L_\phi, \tag{16}
\]

Hence, the current flowing through the short circuit can be expressed as follows:

\[
L_{\text{Ph-Ph}} = 2 \cdot (a - a^2) \cdot L_{\text{L}} - (a - a^2) \cdot L_\phi, \tag{17}
\]

As it can be observed from this description, the current \(L_{\text{Ph-Ph}}\) is dependent on two components. This leads to the disadvantage that a complete reduction of the system homogeneity degree is not possible anymore. The ratio between both components

Fig. 5. Different multi-phase faults with fault resistance

Fig. 6. Phase-phase fault with fault resistance (representation in symmetrical components)
plays a big role and the compensation angle cannot fully eliminate the influence of system non-homogeneity.

In this case it is assumed that the negative sequence current is significantly higher than the zero sequence current. As a result the same approximation of the current \( I_F \) and compensation quantity as in the case of the phase-phase fault can be applied. To apply the same calculation approach for the phase-phase fault with and without earth participation (the same loop type) the following expression is introduced:

\[
X_{\text{Fault}} = \frac{\sin\phi \cdot \text{Im}[U_{\text{Ph-Pe}} \cdot L_{\text{Ph-Pe}}]}{\text{Im}[\exp^i\cdot L_{\text{Ph-Pe}} \cdot I_{\text{Comp}}}]
\] (18)

\[
R_F = \frac{\text{Im}[U_{\text{Ph-Pe}} \cdot Z_i \cdot L_{\text{Ph-Pe}}]}{\text{Im}[2 \cdot (a - a')L_{\text{A,3}} - (a - a')L_{\text{L,i}} \cdot Z_i \cdot \Delta Z_{\text{Ph-Pe}}]} \]

Considering the symmetrical three-pole fault it can be concluded that the load decoupling with zero- and negative-sequence components is not possible. For this goal the superposition principle, in which the contribution of the fault current can be represented in the form of the so called delta-component, is very practical:

\[
L_F = \Delta L_1,
\] (19)

where \( \Delta L_1 \) is the delta-component as difference between pre-fault and fault condition.

The impedance calculation is based on following formula:

\[
X_{\text{Fault}} = \frac{\sin\phi \cdot \text{Im}[U_1 \cdot L_{\text{Comp}(1\text{Ph})}]}{\text{Im}[\exp^i\cdot L_1 \cdot I_{\text{Comp}(1\text{Ph})}]},
\] (21)

where \( U_1 \) and \( I_1 \) are positive sequence voltage and current respectively. \( Z_i \) denotes impedance in positive sequence.

As can be seen here, the impedance calculation with reactance method and fault resistance separation can be applied for each loop so that a full scheme distance protection is possible.

**IV. ANALYSIS OF METHOD**

In order to demonstrate the method behavior three main scenarios were investigated: load flow, power swing and typical faults with fault resistance under load flow.

**A. Pure Load Flow**

For the typical load condition it is required that the impedances of the loops are located far away from the pickup area. It is extremely dangerous if these measured impedances come into the pickup area because this can contribute to unintended trips. The classical method is not robust enough for the high load flow as presented in Fig. 9.

![Fig. 7. Phase-phase-to-earth fault with fault resistance (representation in symmetrical components)](image1)

![Fig. 8. Three phase fault with symmetrical fault resistance (superposition principle)](image2)
All impedances computed with the classical method are located in the protected zone and contribute to the pickup of the distance element. In order to eliminate this unintended effect the so called load encroachment characteristic must be applied for cutting out the affected zone area. It makes the distance element more complex especially in situations where the load flow is not a deterministic value. In case of the reactance method the fault resistance area is separated from the load flow area. Thus, during the pure load flow an extremely high value as “fictive” \( R_f \) is calculated and the loop impedances are not close to the pickup area (Fig. 10). The pickup during pure load flow cannot occur with distance reactance element so that here no additional encroachment characteristic is necessary.

B. Power Swing

The power swing phenomenon has a big influence on the distance protection behavior. This phenomenon can contribute as well to a protection pickup. During power swing the impedance is moving into the pickup area. As a consequence, the so called power swing blocking mechanism is applied in parallel to distance elements in order to ensure the protection security [8]. The calculated impedances with conventional method during power swing are shown in Fig. 11. All impedances trajectories pass through the pickup area so that without power swing detection logic the pickup could not be avoided.

Since the power swing is related to the load fluctuation the reactance method separates this phenomenon from fault condition as well. It is shown in Fig. 12. Due to this fact the application of the power swing blocking mechanism is not necessary in most applications. The usage of distance element with reactance method simplifies the protection algorithm in a high manner.

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**Fig. 9.** Computed impedance trajectory of the classic impedance method during typical high load flow (top: phase-to-earth loops; bottom: phase-to-phase loops)

**Fig. 10.** Computed impedance trajectory of reactance method during typical high load flow (top: phase-to-earth loops; bottom: phase-to-phase loops)

**Fig. 11.** Computed impedance trajectory of the classic impedance method during instable power swing (top: phase-to-earth loops; bottom: phase-to-phase loops)

**Fig. 12.** Computed impedance trajectory of the reactance method during instable power swing (top: phase-to-earth loops; bottom: phase-to-phase loops)
During the fault with resistance an additional voltage drop in the faulty loop appears. As mentioned above, this effect has a significant impact on the reactance result if the calculation of impedance is performed with conventional method. Before, it was assumed that load flow during multi-phase faults has not big influence on the measured reactance. The results of this simulation as well as some practice cases cannot confirm this assumption so that the application of the reactance method for the multi-phase is necessary. Under load flow condition the phase-phase fault CA with fault resistance was simulated and the impedance was calculated for all possible loops. The real fault reactance is 28Ω (Fig. 13). The computed reactance from the classical method is 41Ω and hence significantly higher than the expected value. The resulting reactance deviates from the expected value about 13Ω. It exhibits high under-reach which is not acceptable. In this case the protection algorithm is not selective enough. Such fault condition is tripped after longer time or by backup protection. The reactance method computes correct reactance, so that over-/underreach effects are eliminated. Moreover, the loops that are not involved in the fault are also very remote from the faulty loops (Fig. 14).

V. SUMMARY

In this paper it was described how impedance measurement with reactance characteristic and resistance separation can be applied for multi-phase faults. The unintended effect of impedance moving into the pickup area during pure load conditions is eliminated. The compensation signal applied in the equations leads to an improved reactance result and has positive influence on the selectivity and security of the distance element.

REFERENCES