# Impact of correlated infeeds on risk-based power system security assessment

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***Abstract*—In this paper we investigate how the detailed modelling of uncertainty impacts the assessment of the power system security. We propose a formulation for the inclusion of uncertainty in power system operational security assessment. The model proposed consists of a Monte-Carlo (MC) framework which allows to capture the complex stochasticity of the system inputs using the copula theory for the sampling of the system infeeds, in combination with AC power flow computations for a detailed assessment of the system operation. Taking into account the infeed forecast uncertainty (loads and renewable generation), and the set of possible contingencies, the MC formulation allows the detailed evaluation of the network operation by computing the system risk in terms of Lost Load. Moreover, the model computes the probability of cascading events, pointing out which remedial actions are to be expected and the associated amount of Lost Load. The proposed model is applied to the IEEE 118 bus test system. To highlight how uncertainty modelling impacts the results, we investigate how correlated infeeds affect the severity of estimated system stress. Results show that applying simplifications such as assumptions of independence or approaches that rely on DC approximations could lead to an underestimation of the system risk.**

**Keywords: power system security, Monte-Carlo simulation, cascading events, AC power flow, stochastic inputs, stochastic dependence, correlation.**

## Abbreviations

| | |
|---|---|
| AC | alternating current |
| AGC | automatic generator control |
| cdf | cumulative density function |
| DC | direct current |
| DLF | deterministic load flow |
| FACTS | flexible AC transmission system |
| GPU | graphics processing unit |
| HPC | high performance computing |
| HVDC | high-voltage DC |
| MC | Monte Carlo |
| OL-RBSA | online risk-based security assessment |
| OPF | optimal power flow |
| pdf | probability density function |
| PF | power flow |
| PLF | probabilistic load flow |
| RES | renewable energy source |
| rv | random variable |
| TSO | transmission system operator |
| VR | vulnerability region |

## Nomenclature

| | |
|---|---|
| $\mathcal{A}$ | event |
| $C$ | copula function |
| $F_{X_i}$ | marginal probability density function |
| $F_{X_1 \ldots X_n}$ | cumulative distribution function |
| $\hat{\mu}$ | sample mean |
| $\mu_k$ | average load at bus $k$ |
| $N$ | number of MC samples |
| $N(\mu_k, \sigma_k)$ | normal distribution function |
| $n_b$ | number of buses |
| $P$ | active power |
| $\hat{p}$ | estimated probability |
| $R$ | correlation matrix |
| $\rho$ | Pearson's product-moment correlation |
| $\rho_r$ | rank correlation |
| $s$ | sample standard deviation |
| $\sigma_k$ | standard deviation at bus $k$ |
| $U_i$ | uniform random variable |
| $X_i$ | random variable |

## I. Introduction

Secure grid operation in electrical power systems becomes increasingly more complex. Transmission system operators (TSOs) face a higher uncertainty in system operation due to higher shares of renewable energy sources (RES) and due to the operation in a competitive market environment. In order to ensure security of the system it is therefore of high importance to include uncertainty in security assessment and in the day-ahead operational planning [9], [11], [12].

Two main sources of uncertainty in power system analysis are (i) input uncertainty, i.e., the uncertainty in load and generation, and (ii) configuration uncertainy, i.e., uncertainty related to outaged components. The probabilistic load flow (PLF) formulation was the initial framework for including uncertainty in power system steady-state analysis. Mainly due to the limited computational power, an analytical PLF approach was initially proposed by Borkowska [3] in the 1970s. The method was further developed by Allan and others, see e.g., [1], [2]. The analytical approach is based on convolutions of the involved probability density functions (pdfs). To reduce the complexity of these computations, analytical PLF comes with the following key simplifying assumptions (one or more applying to each formulation), namely: (i) the steady-state model is linearized around an operating point, (ii) the system inputs are assumed to be normally distributed, and (iii) the system inputs are assumed to be statistically independent [4].

The main alternative to analytical approaches is PLF based on Monte-Carlo (MC) simulation. The MC framework does not rely on such simplications as the approach comes down to computing a deterministic load flow (DLF) many times. For this reason, the method has a very high accuracy and can be used as a reference to assess approximate methods. However, MC typically requires a large number of samples, thus many DLFs, and is therefore computationally intensive.

The modelling of wind power has been widely implemented in analytical PLF frameworks with several approaches focusing on the improvement of the accuracy of the approximations, see e.g., [7], [22], [23]. However, it is acknowledged that in order to capture the complex spatial dependency between wind infeeds, an MC framework based on the copula theory is necessary [16]. In [18] the copula theory is used to generate the system inputs for an MC based PLF methodology to investigate large-scale integration of 5 GW of wind power on the system load in the Netherlands. In [17] the proposed model is used to analyse spatial interdependence structures of prediction errors and to assess a 2.1 GW power system in western Denmark.

In this paper an MC based PLF model is proposed for power system operational security assessment which captures infeed forecast uncertainty in great detail. It will be shown that dependence between system inputs cannot be neglected as this would lead to severe underestimation of the system risk.

The paper is organized as follows. In Section II the dependence between system inputs is investigated. In Section III the copula theory is discussed. In Section IV a brief review is provided on other state-of-the-art risk-based modelling, followed by a discussion on the proposed model for system security. In Section V the experimental setup for a case study on the IEEE 118 bus test system is presented. The results from this study are presented in Section VI. Conclusions and future research follow in Section VII.

## II. Nodal dependence structure

Data analysis confirms that wind power infeeds are spatially correlated. In Figure 1 the correlation matrix of wind power forecast errors is shown for a case study in Northern Germany. This case study was part of Work Package 2 of the FP-7 Umbrella project and presented at WIPFOR 2013[1]. In the study wind power infeeds at 29 grid nodes are examined of which 7 wind farms are within the range $200 < P \leq 1000$ MW, 8 wind farms within the range $100 < P \leq 200$ MW and 14 smaller wind farms with $P \leq 100$ MW. The matrix is sorted to the distance to a predefined bus 1.

[1] See "Probabilistic wind power forecasting in transmission grids – making use of spatial correlation" at: http://conferences-osiris.org/wipfor.

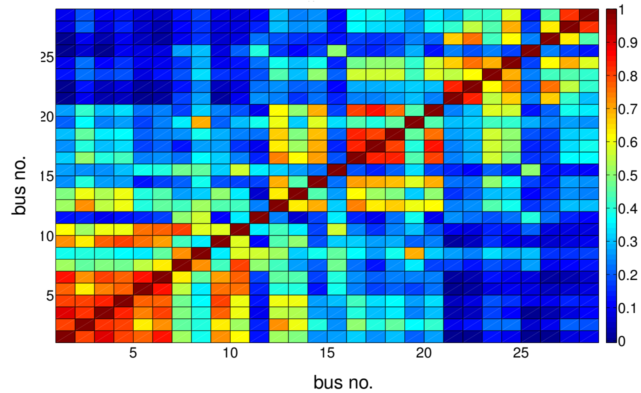


Fig. 1. Correlation matrix of wind power forecast errors.

The figure shows that:

- Wind power infeeds at neighbouring grid nodes can be strongly correlated (e.g., see the red lower left corner);
- The correlation varies strongly with the distance between the grid nodes.

In case of statistical independence (correlation = 0) the figure would have been entirely blue with a red diagonal only.

## III. Copula theory

In numerical simulation, copulas are a powerful tool to generate multivariate distributions with given dependence structure regardless of the marginal distribution functions (marginals). In particular, they are useful to capture stochastic phenomena of which the marginals are non-Gaussian [19]. Sklar introduced (1959) copulas as functions that "join together one-dimensional distribution functions to form multivariate distribution functions" [21]. More precise, copulas are multivariate distribution functions whose one-dimensional marginals are uniform on the interval $[0, 1]$ [13].

### A. Sklar's theorem

Sklar's theorem [21] states that for all cumulative distribution functions $F_{X_1 \ldots X_n}(x_1, \ldots, x_n)$ with one-dimensional marginals $F_{X_i} (i = 1, \ldots, n)$ there exists a copula $C$ such that

$$F_{X_1 \ldots X_n}(x_1, \ldots, x_n) = C\big(F_{X_1}(x_1), \ldots F_{X_n}(x_n)\big).$$

Moreover, if all $F_{X_i} (i = 1, \ldots, n)$ are continuous, then $C$ is unique.

### B. From marginals/data to uniforms/ranks

A random variable (rv) $X$ is transformed into a uniform rv $U$ on $[0, 1]$ by applying the cumulative distribution function (cdf) $F_X$. Likewise, by applying the inverse cdf $F_X^{-1}$ (provided that the inverse exists), a uniform rv $U$ on $[0, 1]$ can be transformed into a random variable $X$, i.e.,

$$U = F(X) \iff X = F_X^{-1}(U).$$

In case $F_X$ is not available, the empirical cdf may be used.

To transform data from distributions to ranked variables (ranks) the data is sorted and the respective values are replaced by their positions in the sorted list.

## C. Rank correlation

Since Pearson's well-known product-moment correlation coefficient, usually denoted by $\rho$, is not invariant under general cdfs, Spearman's rank correlation coefficient is used instead. Spearman's correlation coefficient is defined as Pearson's correlation coefficient, denoted by $\rho_r$, applied to the ranked variables. The rank correlation $\rho_r$ of rvs $X_i$ with cdfs $F_{X_i}$ is thus defined as

$$\rho_r(X_1, \ldots, X_n) := \rho\big(F_{X_1}(X_1), \ldots, F_{X_n}(X_n)\big).$$

The rank correlation measures the monotinicity between rvs.

## D. Sampling

The sampling of the system inputs is done as follows:

1) Analyse the stochastic dependence in the uniform/rank domain. From this analysis extract a correlation matrix $R$;
2) Generate correlated ranks $U_i (i = 1, \ldots, n)$ from the standard uniform distribution using a copula model and correlation matrix $R$;
3) The correlated ranks $U_i$ are transformed into marginal distributions by applying $F_{X_i}^{-1}$ to $U_i (i = 1, \ldots, n)$.

# IV. Risk-based Power System Security Assessment

## A. State-of-the-art modelling

In the last years considerable efforts have been devoted to power system risk or reliability assessment. In this section a brief overview is given of the latest developments.

In a comprehensive list of papers [8]—[10], [20] Kirschen *et al.* focus on various aspects of risk-based security assessment, including random contingencies, cascading events, sympathetic tripping, hidden failures and weather effects. In these papers, an MC simulation approach is advocated to capture the complex nature of random disturbances and rare events. Since the focus is on the operational environment, the MC framework is used for sampling of the random disturbances and not for capturing load forecast uncertainty or weather conditions. The MC model includes AC power flow (PF), heuristic load shedding and corrective actions. In [9] a quantitive comparison between the proposed risk-based approach and a traditional, determinstic approach is provided. In [10], [20] the phenomenon of hidden failures in networks is treated in more detail using the notion of vulnerability regions (VRs).

Another main stream of research is performed by McCalley and co-workers. Various risk-based security measures are presented in [5], [14], [15], [24] and [25]. In particular, in [14] and [15] Ni and McCalley propose an approach based on so-called risk indices which allows control room operators to assess the security levels of power systems in real-time. The method is referred to as online risk-based security assessment (OL-RBSA). In contrast to Kirschen's approach, a predefined list of contingencies is used. So-called severity functions are used to capture risk (against low voltage, circuit overload, voltage instability and cascading). Cascading overloads are modelled with a looping mechanism based on hard limits.

## B. The proposed model

To investigate the impact of correlated infeeds on risk-based power system security assessment the following model is used. The model joins features of the methods described in the preceding section, namely:

- similar to Kirschen an MC simulation framework is used which allows to capture the complex stochasticity of the system inputs and the highest level of detail. Initiating conditions that have low probability but high risk can be simulated;
- full AC PF computations which provide the highest level of accuracy and which allow for detection of voltage problems. DC approximations of the power flow problem cannot detect voltage problems;
- In this model cascading events are modelled similar to McCalley's approach. A typical simple criterion for cascading is that circuits are overloaded by 125% to 150% depending on the weather conditions;
- The infeed uncertainty is modelled by using the copula theory, see Section III.

The focus in this paper is on day-ahead rather than on real time simulation for systems with high levels of RES, hence high forecast uncertainty. Therefore, the infeeds are sampled rather than the contingencies. However, sampling of contingencies and inclusion of sympathetic tripping is very straightforward in the flexible model.

The primary output of the model is expected Lost Load [MW]. Additionally, the model can provide the expected amount of shifted generation [MW], the probability of four levels of severity:

1) none;
2) redispatching required;
3) redispatching + load shedding required;
4) system collapse,

and the probability of overloaded circuits, voltage violations, and cascading events. Both load shedding and system collapse contribute to Lost Load; in case of a system collapse it is assumed that all connected load is lost.

## C. Model description

Consider Figure 2 which shows the flowchart for 1 MC trial. Given the topology of the system, an initial dispatch and a post-contingency state, for each MC sample a different stochastic load profile is generated using the copula theory, see Section III-D. First, it is verified how many subsystems there are and, if required, slack buses are added in order to be able to solve the PF equations mathematically. Next, a full AC PF computation is performed. If the AC PF does not converge, it is assumed that this indicates a severe system state and correcting actions such as redispatching are needed. If the AC PF does converge, it is checked whether there are heavily overload circuits ($> 1.5\times$ continuous rating). If so, these circuits are removed from the system. This models immediate and automatic tripping. The process may repeat itself (the indicated loop) until there are no heavily overloaded ciruits anymore. The model thus allows for capturing cascading

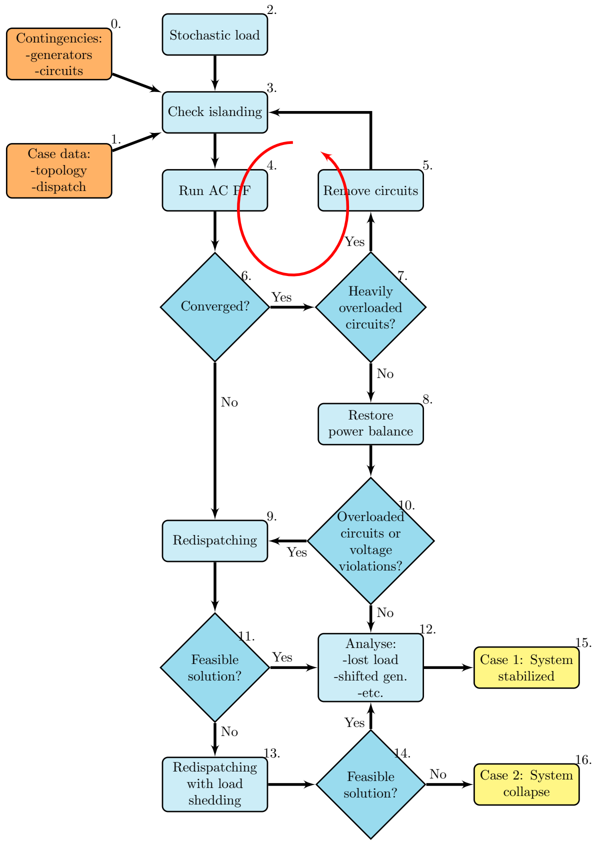


Fig. 2. Flowchart for 1 Monte Carlo sample.

events and system islanding. After the cascading loop the power balance is restored using automatic generator control (AGC). If after AGC there are still overloaded circuits or voltage violations at the buses, correcting actions are required to stabilize the system. First it is verified if redispatching generation can resolve the problems; if not, redispatching in combination with load shedding is applied. To determine how to redispatch generation and how and where to shed load, an extended AC OPF is used. Herein load shedding is modelled by dispatchable loads. A clever usage of the extended AC OPF allows for finding a new economic dispatch which deviates minimally from the original dispatch in which all voltage violations and circuit overloads are removed. The penalties for load shedding are set to high values such that load shedding is used only as a last resort.

In future versions of the model other measures such as topology optimization, inclusion of flexible AC transmisson systems (FACTS) and high-voltage DC (HVDC) lines may readily be implemented.

### *D. Monte-Carlo framework*

For the uncertainty analysis a Monte-Carlo (MC) simulation framework is adopted which allows to capture the complex stochasticity of the system inputs, and enables the mapping of specific events with catastrophic consequences. The uncertainty analysis module provides samples of the possible system states, taking into account the infeed forecast uncertainty (loads and renewable generation). The MC framework allows the detailed representation of the input uncertainty, enabling the modelling of correlated non-normal infeed distributions as is the case of wind forecast uncertainty [17] and the detection of system states with low probability but high risk. As usual the probability of an event $\mathcal{A}$ is estimated as

$$\hat{p} = \frac{\text{The number of occurences of event } \mathcal{A}}{N}, \qquad (1)$$

where $N$ is the number of MC samples. The 95% confidence interval is given by

$$\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{N}}, \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{N}}\right). \qquad (2)$$

Similarly, the 95% confidence interval for a quantity (e.g., Lost Load, redispatched generation) is given by:

$$\left(\hat{\mu} - 1.96\frac{s}{\sqrt{N}}, \hat{\mu} + 1.96\frac{s}{\sqrt{N}}\right), \qquad (3)$$

where $\hat{\mu}$ is the sample mean, $s$ the sample standard deviation and $N$ the number of MC samples.

### *E. Full AC computations*

The AC framework allows the detailed detection of voltage problems which cannot be captured with approximate solutions of the power flow problem, such as a DC approach. Voltage problems are often the generating causes of system collapse. It is recognized that full AC PF computations entail a higher computational burden which has to be taken into account for designing an operative solution. By speeding up the AC PF computations, e.g., by deploying modern Newton-Krylov iterative solution methods [6], the overall computation time can strongly be reduced. In Section VII more key approaches are discussed to tackle the issue of computational intensity.

### *F. Stochastic load*

The stochastic load at the buses is generated according to the procedure given in Section III-D.

## V. Study Case - Experiments

In this section we will analyze the impact of forecast uncertainty to the power system security assessment and investigate how key modelling assumptions affect results. We focus on the impact of the two key aspects of forecast uncertainty: (i) the accuracy of day-ahead forecasts and (ii) the correlation between forecast errors. For this, we design experiments where we assess the system security for varying levels of these aspects and present the impacts in a condensed way by means of iso-risk plots[2]. The experiments are therefore set up as follows. An initial (post-contingency) state of the system is chosen, consisting of an initial dispatch and network topology (list of contingencies). We investigate 110 setups as follows: (i) forecast uncertainty ranging from 0 (perfect forecasts) to 100% (day-ahead forecast uncertainty as measured in TSO data) in steps of 10% and (ii) forecast error correlation ranging from $\rho_r = 0$ (independent infeeds) to $\rho_r = 0.9$ (highly dependent infeeds) in steps of 0.1. For each setup an MC simulation with

[2]An iso-risk plot is a map in which points of equal risk are connected by contour lines.

10.000 samples is performed using the same initial system state where specific metrics of system security are measured (see Section IV): the Lost Load, the probability of four levels of severity, and the probability of specific events.

Next, an equidistant 2D-grid is defined consisting of $11 \times 10$ grid points which are used to construct iso-risk plots of the security metrics. Along the $x$-direction is the forecast uncertainty and along the $y$-direction is the correlation between the load buses.

### A. *Implementation and hardware*

The presented model is implemented in MATLAB using the MATPOWER package [26]. The MC simulation is performed in parallel on the four physical cores of a 3.4 GHz Intel Xeon E3-1240 processor.

### B. *Test system*

For the numerical experiments the IEEE 118 bus test system is chosen. This system consists of 118 buses (91 load buses), 54 generators (thermal units) and 186 circuits (9 transformers). Although the 118 bus network is a default test case in MATPOWER, slightly different and more complete data is downloaded from http://motor.ece.iit.edu/data/JEAS_IEEE118.doc. The data is used to update the standard MATPOWER 118 bus case file. Moreover, in order to observe higher system stress, the following modifications are made: (i) the load level is increased to 120%, (ii) the continuous line ratings are set to 90%. In the modified system the total active load is 4481 MW, the total reactive load is 1732 MVar and the total available generation capacity is 7220 MW.

### C. *Initial dispatch and topology*

The initial dispatch is determined by running a default AC OPF, i.e., minimization of generation costs. Two network topologies are studied:

1) the system without any contingencies (default topology);
2) the system in which two arbitrary circuits are outaged at the same time, namely circuits 23 and 144 (weakened topology).

### D. *Load forecast uncertainty*

Although the presented model allows to work with non-normal distributions, in this paper the load is considered to be normally distributed. At each bus $k = 1, \ldots, n_b$ the load is captured by $N(\mu_k, \sigma_k)$, where $\mu_k$ is the average load, given by the corresponding number in the case file. The forecast uncertainty is modelled choosing the magnitude of the standard deviation $\sigma_k$ according to the results of the statistical analysis of typical day-ahead forecast errors from European TSOs (the analysis was based on the output of Work Package 2 of the FP-7 Umbrella project). The data analysis shows that the normalised standard deviation $\sigma_k/\mu_k$ ranges between $0.34$ and $1.36$ (most fractions are around $0.40$), indicating the high levels of uncertainty on which TSOs are typically exposed to.

### E. *Correlation matrix*

In a real-time application the correlation matrices should be build up based on constant updates of historical meteorological data in order to capture phenomena as wind direction and wind speed, and on geographic data such as distance and orography. In [18] it is discussed how this can be done. The focus of this paper is rather to show that it is necessary to include stochastic dependence, for which a uniform correlation matrix already suffices. The load at the buses is considered to be uniformly correlated, i.e., the correlation matrix $R \in \mathbb{R}^{n_b \times n_b}$ is given by

$$R = \begin{bmatrix} 1 & \rho_r & \rho_r & \cdots & \rho_r \\ \rho_r & 1 & \rho_r & \cdots & \rho_r \\ \rho_r & \rho_r & 1 & \cdots & \rho_r \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_r & \rho_r & \rho_r & \cdots & 1 \end{bmatrix},$$

where $\rho_r$ is the rank correlation and $n_b$ the number of buses. In this study $n_b = 118$. Using this stochastic dependence setup allows to obtain results on the impact of correlation. However, it is emphasized that the proposed method can be applied using any correlation matrix, once available.

### F. *Sampling of the system inputs*

The sampling of the system inputs is done according to the procedure sketched in Section III-D. In this study the correlated ranks are obtained by applying the Gaussian copula model with the correlation matrix $R$ as given in Section V-E. Thereafter, the correlated ranks are transformed into marginal distributions by applying the inverse cdf of the normal distribution with $\mu_k$ and $\sigma_k$ as described in Section V-D. Although in this paper only Gaussion marginals and a Gaussian copulasare studied, the presented method can model any type of marginals and copulas. In [18] it is indicated how realistic marginal distributions can be calculated based on historical data.

## VI. RESULTS AND DISCUSSION

In Figure 3 the iso-risk plot is presented for Case 1 (no contingencies). The iso-risk curves are defined by the positions in the $xy$-plane with the same expected amount of Lost Load. On the $x$-axis is the load forecast uncertainty, whereas on the $y$-axis is the correlation between the load buses.

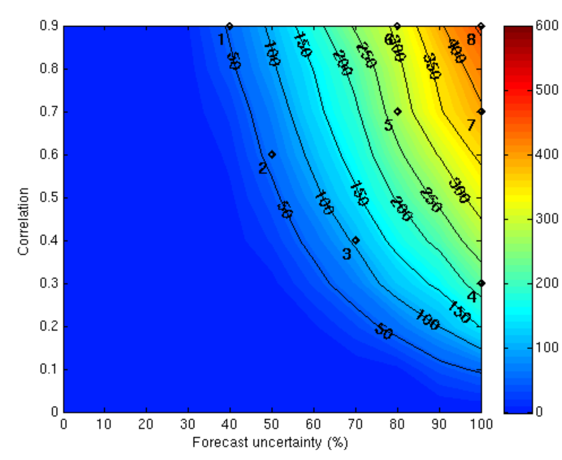


Fig. 3. Iso-risk plot for uniform correlation matrix in case of no initial contingencies.

The figure shows the significant impact of uncertainty in the system security assessment and that not taking into account correlation between the load at the buses (e.g., wind power forecast errors) would lead to severe underestimation of the system risk. Although for low levels of forecast uncertainty (0 to 30%) this poses no significant difference, when the forecast uncertainty is increased (30% and above), the Lost Load is significantly higher when correlation is taken into account. The analysis indicates that methods that assume independence ($\rho_r = 0$) would not detect any system stress while in reality the system could experience significant levels of Lost Load.

In Figure 4 a similar iso-risk plot is presented for Case 2 (two arbitrary outaged circuits). As can be seen, the expected amount of Lost Load is higher because of the weakened system. However, the overall risk pattern remains the same; the highest risk is observed for high forecast uncertainty and strong dependence between the load at the buses.

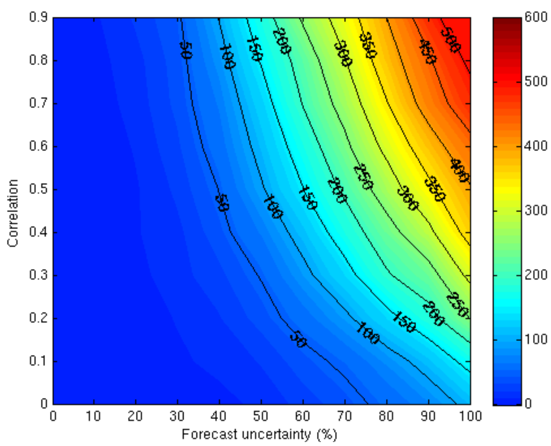


Fig. 4. Iso-risk plot of expected Lost Load for uniform correlation matrix in case circuits 23 and 144 are outaged.

The presented approach can be used for screening the power system and for predicting severe operating states. Such plots can be used in the day-ahead system planning to assess operational actions to steer the system in risk-averse operating modes.

From the data it is observed that the standard deviation of the metric increases with the amount of Lost Load. For the most risky operating conditions (forecast uncertainty = 100%, correlation = 0.9) the sample standard deviation is 922 MW$^2$ for Case 1. The 95% confidence interval for the expected amount of Lost Load is $461 \pm 18$ MW according to Equation (3). For Case 2 the accuracy is the same.

As an additional output, the model can investigate the different levels of system severity, recall Section IV. In Figure 5 for Case 1 these four levels of severity and corresponding probabilities are shown for eight different operating conditions in the $xy$-plane, indicated with markers ($\diamond$) in Figure 3. See Table I for their precise location. The probabilities of occurrence are estimated by enumeration of the four levels, see Equation (1). The figure shows that the higher the forecast uncertainty and/or the higher the correlation between the load at the buses, the higher the probability that serious correcting measures are needed to stabilize the system. It can also be seen from the figure that a combination of high forecast uncertainty

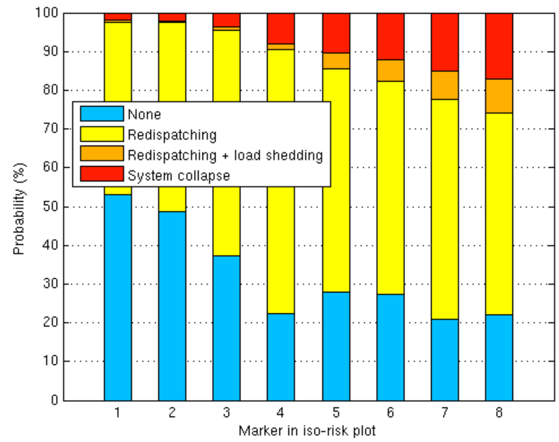


Fig. 5. Four levels of severity and corresponding probabilities.

| Marker | Forecast uncertainty (%) | Correlation |
|---|---|---|
| 1 | 40 | 0.9 |
| 2 | 50 | 0.6 |
| 3 | 70 | 0.4 |
| 4 | 100 | 0.3 |
| 5 | 80 | 0.7 |
| 6 | 80 | 0.9 |
| 7 | 100 | 0.7 |
| 8 | 100 | 0.9 |

TABLE I. MARKER LOCATIONS

and high correlation results in a high probability of system collapse. The accuracy is high for large probabilities: e.g., for Marker 6 the 95% confidence interval of the probability of system collapse is given by $0.0987 \pm 0.0058$ according to Equation (2). However, for small probabilities (rare events), the accuracy is lower. E.g., for Marker 1 the 95% confidence interval of the probability that redispatching and load shedding is required is $0.0046 \pm 0.0008$.

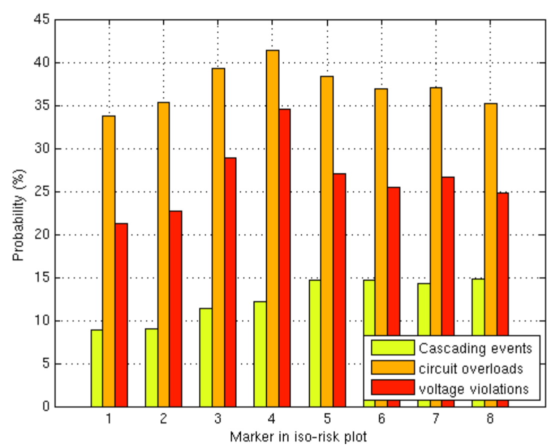


Fig. 6. Probability of cascading events, overloaded circuits and voltage violatons.

It is also interesting to analyze the composition of the system risk. In Figure 6 the probabilities of cascading events, overloaded circuits and voltage violations are presented for the operating states indicated by the markers. As can be seen, cascading is more likely in cases of higher forecast uncertainty and/or higher correlation, while besides circuit overloads, voltage violations contribute heavily to the system risk. In this respect, DC OPF based methods would underestimate the risk since they do not allow capturing voltage events.

Finally, a short note on the computation time. The overall computation time to run 110 times a 10.000 samples MC simulation was approximately 1 day on four cores of a regular computer, see Section V-A.

## VII. CONCLUSIONS AND FUTURE RESEARCH

In this paper a methodology was presented to analyze the impact of correlated system infeeds on risk-based security assessment. The proposed model which combines an MC simulation framework, full AC PF computations and the copula theory for sampling of the system inputs allowed for a detailed investigation of the impact of uncertainty to the system security. A case study of the IEEE 118 bus test system showed that not taking into account correlation between the load at the buses may lead to severe underestimation of the system risk. Also, it was shown that voltage problems contribute heavily to severity. Therefore, methods that rely on DC approximations may lead to inaccurate predictions of the system risk as well.

A combination of an MC framework and full AC PF computations entail a higher computational burden. Below key solutions to resolve this issue are discussed. Firstly, it must be noted that the model was implemented in MATLAB in combination with the MATPOWER package. However, by implementing the model in a programming language which supports high performance computing (HPC) the overall computation time can strongly be reduced. To massively parallelize the MC simulation one may think of MPI to address multiple computers or, more recently, CUDA to take advantage of GPU computing. Secondly, adjoint techniques may be used to accelerate non-linear gradient-based optimization, such as AC OPF computations. Thirdly, AC PF computations can be speed up by deploying modern Newton-Krylov methods since the use of iterative solution techniques is very suitable for data reuse in the repeated PF computations in MC and for contingency analysis. Finally, variance reduction techniques based on rare event simulation can be used to reduce the required number of MC samples.

These techniques will be subject of future research. Also interesting is to analyse the impact of correlated infeeds using non-normal distributions, e.g., beta distributions, to model wind power. Finally, the application of the proposed method to a real case is in the plans for future work. This should be straightforward since in the proposed methodology there are no constraints to the type of distributions used.

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