Abstract— This paper presents a general algorithm to calculate an optimal time step size and a maximum simulation time for EMTP-based programs. This is of particular importance for new users of EMTP-based programs, since the user is responsible for setting up these parameters before running a simulation case. The selection of the time step size affects the precision of the simulation. The time step size depends on the maximum frequency expected in the phenomena, which is normally unknown, a priori. A robust algorithm is presented here based on all the input data given for the circuit under simulation. The proposed calculation process is based on single- or multi-phase uncoupled or coupled circuits, with lumped or distributed parameters. Simulations are given demonstrating the effectiveness of the proposed rules. A future challenge will be the creation of a methodology capable of adapting the time step size dynamically.

Keywords— EMTP, Time step size, Accuracy of discrete-time solutions, Computer modelling, Time constant, Eigenvalue.

I. INTRODUCTION

The existing versions of the EMTP software [1]-[7] do not provide the user with an optimal estimate of the time step size (\( \Delta t \)) to be used for a given simulation case. It is up to the user to choose and adjust it properly. The same situation happens for the maximum simulation time (\( t_{max} \)).

Even though there is a close relationship between the time step size, the time constants, the propagation time, and the oscillations periods of voltages and currents in a simple circuit, this relationship is difficult to verify for large systems, because usually there is no prior knowledge of the analytical solution.

Programs like SPICE [8] change the time step continuously during the simulation, according the slope of change of the circuit variables. This strategy, however, can result in increased solution times and in some cases in non-convergence [9] or numerical instability conditions.

The methods described in this paper, follow the traditional EMTP approach of using a constant time step calculated according to the maximum frequencies that need to be simulated accurately.

Unless the circuit equations are solved analytically (or the eigenvalues are somehow calculated for linear systems or for linearized parts of nonlinear systems [6]-[7]), the time parameters are not known by inspection. It is possible, however, to establish “ranges” in which the time step size and the maximum simulation time must be contained. This may help new and experienced EMTP-based program users to minimize the simulation time without compromising the accuracy of the results. In this case, the main focus of the study can remain on the analysis of the transient system response, rather than on a trial and error approach to adjust the time step size and the maximum simulation time by running multiple simulations.

In principle, it is well known for EMTP-based simulation that the maximum frequency expected to be present in the simulation (2) should be at least five times less than the Nyquist frequency (1).

This provides a reasonable accuracy (error in the order of 3.31%) in the modelling of the discretized lumped electrical components (inductor and capacitor) using the trapezoidal rule of integration [1]-[2]. This rule results in (3) for the calculation of the time step size \( \Delta t \).

\[
\frac{1}{2\Delta t} = \frac{1}{10 f_{Ny}} \quad (1)
\]

\[
f_{max} = \frac{5}{1} \times f_{Ny} = \frac{5}{1} \times \frac{1}{2\Delta t} = \frac{5}{10 \Delta t} \quad (2)
\]

\[
\Delta t = \frac{1}{10 f_{max}} = \frac{T_{min}}{10} \quad (3)
\]

In terms of frequency content, Fig. 1 [10] illustrates the wide speed of phenomena in electrical power systems.

A common practice for EMTP-based simulation is to always adopt a time step size of

\[
\Delta t = 50 \, \mu s
\]

which then implies that:

\[
f_{Ny} = \frac{1}{2 \times 50 \, \mu s} = \frac{10 \, MHz}{100} = 10 \, kHz
\]

\[
f_{max} = \frac{5}{1} \times f_{Ny} = \frac{5}{1} \times \frac{1}{2\Delta t} = \frac{5}{10 \times 50 \, \mu s} = 2 \, kHz = 2 \times 10^3 Hz
\]
If one chooses, for example a $\Delta t$ of 1 microsecond, even though very small, it might not be adequate for example, when simulating very high frequency transients in SF6 gas insulated substations. For the very common $\Delta t$ of 50 microseconds, only transients up to 2 kHz will be properly simulated.

II. RANGES OF TIME CONSTANTS

This paper develops an algorithm to estimate an adequate $\Delta t$ automatically from the circuit. In a first simple approach, we will assume that we have no knowledge of the way the circuit components are connected.

Here we “estimate” a minimum and maximum value for all the R’s combined into a single parallel (minimum) or series branch (maximum); similarly for all the L’s and all the C’s, according to (7)-(12). For $n$ R’s, L’s, or C’s we have:

$$R_{MIN} = \frac{1}{n_1 + n_2 + \cdots + n_n}$$  \hspace{1cm} (7)

$$L_{MIN} = \frac{1}{L_1 + L_2 + \cdots + L_n}$$  \hspace{1cm} (8)

$$C_{MIN} = \frac{1}{C_1 + C_2 + \cdots + C_n}$$  \hspace{1cm} (9)

$$R_{MAX} = R_1 + R_2 + \cdots + R_n$$  \hspace{1cm} (10)

$$L_{MAX} = L_1 + L_2 + \cdots + L_n$$  \hspace{1cm} (11)

$$C_{MAX} = C_1 + C_2 + \cdots + C_n$$  \hspace{1cm} (12)

Equations (7), (8) and (12) assume that each type of circuit component is all connected in parallel whereas (9), (10) and (11) assume that each type of circuit component is all connected in series. Correspondingly, we can estimate the maximum and minimum values for inductive or capacitive time constants (i.e., $(T_L)_{MAX}$, $(T_L)_{MIN}$, $(T_C)_{MAX}$, $(T_C)_{MIN}$) and for the system natural modes of oscillations (i.e., $(T_N)_{MAX}$, $(T_N)_{MIN}$), since a natural oscillation time period is given by:

$$T_N = \frac{2\pi}{\sqrt{LC}}$$  \hspace{1cm} (13)

For transmission lines, the proper characteristic impedance $(Z_c)$ and the propagation time $(\tau)$ for each transmission line are also considered, identifying the $T_{MIN}$ and $T_{MAX}$.

With all the time calculated values sorted in ascending order in a vector $(T)$, the “optimum” time step size and the maximum simulation time can be estimated as in (14) and (15), respectively:

$$\Delta t = \frac{1}{100} \times \text{min} \left( T \right)$$  \hspace{1cm} (14)

$$t_{max} = 5 \times \text{max} \left( T \right)$$  \hspace{1cm} (15)

Observe that, by using 1/100 of the minimum $(T)$ in (14), instead of 1/10 as in (3), the transient response at $t = \Delta t$ is closest to the ideal analytical value for $t = 0$. For some cases with distributed parameters transmission lines, a factor of 10 instead of 5, might be used in (15), which guarantees that, after all wave reflections, steady state conditions are reached.

Next, test cases with simulation results illustrate the application of the method. The procedure can be automated in EMTP-based programs.

III. TEST CASES SIMULATIONS

A. Simple RLC circuit

Fig. 2 presents a RLC single-phase circuit where the minimum and maximum of the series and parallel combination of the circuit parameters were calculated to obtain the integration step size (discrete time $\Delta t$) and the maximum simulation time ($t_{MAX}$).

The goal is to get good simulation results for the voltage and current transient in the circuit. The circuit is excited by a DC source with zero initial conditions, where $R = 100\Omega$, $L = 5H$, $C = 250\mu F$, and $e(t) = 10V$.

The minimum and maximum values of the equivalent R, L and C are calculated from equations (7) to (13):
For a circuit composed of a resistance and an inductance, its time constant can be calculated using the expression (16),

\[ T_L = \frac{L}{R} \]  

(16)

For a circuit composed of a resistance and a capacitance, the time constant can be calculated using the expression (17),

\[ T_C = R \cdot C \]  

(17)

Thus, it is possible to determine the minimum and maximum limits for the max and min time constants, which can be either inductive or capacitive,

\[ (T_L)_{MIN} = \frac{L/\omega}{R/\max} \]  

(18)

\[ (T_L)_{MAX} = \frac{L/\min}{R/\omega} \]  

(19)

\[ (T_C)_{MIN} = R_{\min} \cdot C_{\max} \]  

(20)

\[ (T_C)_{MAX} = R_{\max} \cdot C_{\max} \]  

(21)

From (13), it is possible to calculate the minimum and maximum time values related to the system natural oscillations modes, as presented in (22) and (23),

\[ (T_N)_{MIN} = 2\pi\sqrt{\frac{L_{\min}}{C_{\min}}} \]  

(22)

\[ (T_N)_{MAX} = 2\pi\sqrt{\frac{L_{\max}}{C_{\max}}} \]  

(23)

If the voltage source were sinusoidal, for example \( e_1(t) = E_{\max} \cos(\omega t + \theta) \), then its corresponding time period would have to be considered as well:

\[ T_1 = \frac{2\pi}{\omega} = \frac{1}{f} \]  

(24)

In this case, trapezoidal integration and the Nyquist theorem require a value of \( \Delta t \) of about \( T_1/10 \) [11].

Once the time step limits have been calculated, it is now possible to build a table (Table I) with these limits organized in ascending order.

Since the calculated time values in the previous analysis are only the limits or boundaries, a practical approach would be to choose as the initial optimal time step size \( \Delta t \) one submultiple of the smallest calculated time value in Table I, and as the maximum simulation time \( t_{\max} \), five times the biggest calculated time value in Table I.

<table>
<thead>
<tr>
<th>TABLE I. CALCULATED CIRCUIT TIME VALUES</th>
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<tbody>
<tr>
<td>((T_C)_{MIN})</td>
</tr>
<tr>
<td>((T_C)_{MAX})</td>
</tr>
<tr>
<td>((T_L)_{MIN})</td>
</tr>
<tr>
<td>((T_L)_{MAX})</td>
</tr>
<tr>
<td>((T_N)_{MIN})</td>
</tr>
<tr>
<td>((T_N)_{MAX})</td>
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</tbody>
</table>

Applying (14) and (15) to Table I, results (25) and (26) are obtained.

\[ \Delta t = \frac{1}{100} \cdot \min(T) = \frac{1}{100} \cdot 25ms = 25\mu s \]  

(25)

\[ t_{\max} = 5 \cdot \max(T) = 5 \cdot 222ms = 1.11s \]  

(26)

The effectiveness of this criterion can now be tested by simulating the circuit in an EMTP-based program and analyzing, for example, the voltage and current in all the elements (see Fig. 3, Fig. 4, and Fig. 5).

Observe that, by using 1/100 of the minimum \( T \) in (25) (instead of 1/10 as usually calculated with (3)), the transient response at \( t = \Delta t \) for the voltage across the inductor (Fig. 4) is closest to the ideal analytical value \( v(t) = 10[\sin t] \), for \( t = 0^+ \). This is easily verified in the plotting of Fig. 4, and can also be verified by zooming over the transient period.

Moreover, the initial estimate for \( t_{\max} \) is sufficient to contain the full transient period within the simulation window. This is relevant for engineering overvoltage and overcurrent analysis, when travelling wave phenomena, resonances, or ferroresonances may show up after some elapsed simulation time.

Fig. 3. Resistor voltage and current.
Line 2-4 represents a cable which enters the generator step-up transformer of an underground power plant directly through the bushing. The question to be answered in this case is whether a surge arrester located at the cable terminal 2 in the open air substation can protect the transformer at the end of the cable.

The transformer is represented by its surge capacitance, in parallel with a damping resistance approximately equal to the surge impedance of the transformer winding. The capacitance in 1 represents a capacitive voltage transformer. The nonlinear characteristic of the surge arrester and its breakdown voltage are shown in [12].

If the lightning stroke is assumed to hit the phase conductor directly, 30m away from tower in 5, and if assumed a piecewise linear current impulse with \( t_{\max} = 8 \, kA \) at 1 \( \mu \)s, and \( i = 4 \, kA \) at 50 \( \mu \)s, we obtain the overvoltages and currents shown in Figs. 7, 8, 9.

Fig. 7 shows the voltage in location 1 and 2, while Fig. 8 shows the lightning stroke current wave shape, and the discharge current in the surge arrester.

Fig. 9 shows the voltage at the transformer terminal 4, and at node 3. It can be seen that the overvoltage at the transformer terminal (location 4) is somewhat higher (722.4 \( kV \)) than at the surge arrester (location 2) (640.5 \( kV \)), because of the cable connection in between.

The volt-time characteristic of the 220 \( kV \) insulator string at the tower location 5 can also to be considered as explained in [12]. For \( t_{\max} = 8 \, kA \) there is no flashover across the insulator. For \( t_{\max} = 10 \, kA \) the overvoltage in 5 would have intersected the volt-time characteristic, and a flashover to the tower would have occurred.

All these results were simulated using a \( \Delta t = 0.1 \, \mu \)s (the minimum propagation time), and \( t_{\max} = 10 \, \mu \)s. However, if for simplicity we ignored the surge arrester and the volt-time characteristic that determines possible flashover to the tower, we can apply the suggested algorithm for the calculation of the optimal time step size and maximum simulation time.

Table II presents the calculated time values, in ascending order. Applying (14) and (15) to Table II, the results in (27) and (28) are obtained.

Fig. 10 presents the voltages at the stroke location, and at the transformer terminal (location 4), when not including the surge arrester, and running the simulation for this case with \( t_{\max} = 200\mu \)s. A holistic and practical engineering comprehension of this case is possible by running the EMTP-based simulation considering the whole transient period, i.e. with \( t_{\max} = 200\mu \)s. When the maximum instantaneous values or peak values of each nodal voltage are detected, as presented in Table III, this reveals that the maximum reached overvoltages (up to 1480 \( kV \)), i.e. 1.48 \( MV \) in some nodes) occur as expected just at the beginning of the lightning transient wave propagation. Physical meaning arises when also plotting all the curves. Nevertheless, a good engineering approach is to pay attention before in the modelling of the system, and all the simplifications made in the equivalent circuit for the phenomena under investigation.
Tower with volt-time characteristic of insulator string

Busbar sections A, B, C, D with

Z = 370 Ω, c = 300 m/µs
l = 30m

Z = 370 Ω, c = 300 m/µs
l = 30m

Z = 370 Ω, c = 300 m/µs
l = 150m

Surge arrester
3 Ω 1000 pF

4400 pF

Fig. 6. Line diagram of 220kV substation [12].

Fig. 7. Voltage in [kV] in location 1 and 2 [12].

Fig. 8. Lightning stroke current in [kA] and discharge current in the surge arrester at location 2 [12].

Fig. 9. Voltages in [kV] at the transformer terminal 4, and at node 3 [12].

Table II. Calculated Circuit Time Values

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>(Tc)min</td>
<td>14.7732 ns</td>
</tr>
<tr>
<td>(Tc)ave</td>
<td>100 ns</td>
</tr>
<tr>
<td>(Tc)max</td>
<td>1 µs</td>
</tr>
<tr>
<td>(Tc)max</td>
<td>43.1460 µs</td>
</tr>
</tbody>
</table>

\[ \Delta t = \frac{1}{100} \times \min(T) = \frac{1}{100} \times 14.7732ns = 0.15ns \quad (27) \]

\[ t_{max} = 5 \times \max(T) = 5 \times 43.1460\mu s = 216\mu s \quad (28) \]

Fig. 10. Voltages in [kV] at the stroke location, and at the transformer terminal 4, when not including the surge arrester, during the whole transient period.
where:

\[ K_{e_{33}} = 1 \]

\[ K_{e_{11}} = \text{branch parameters according to branch nature, i.e., } \begin{cases} 1 & \text{for inductive branches,} \\ -1 & \text{for capacitive branches,} \\ 0 & \text{for resistive branches.} \end{cases} \]

\[ \Delta t = \text{time step size;} \]

\[ [I] = \text{unit matrix, where all elements are zero, except the diagonals, which are equal to 1;} \]

\[ [A^d] = \text{continuous-time state-space matrix;} \]

\[ [A^c] = \text{continuous-time state-space matrix;} \]

\[ v_i = \text{eigenvector associated with the eigenvalue with index } i; \]

\[ z_i = \text{discrete-time eigenvalue of index } i \]

\[ \lambda_i = \text{continuous-time eigenvalue of index } i; \]

The relevance of this formulation is that the dynamics of the system can be determined for every event, and if desired, at every instant of the simulation, as the topology changes by switches operations, or as we change regions in non-linear elements.

Additionally, fast and slow subsystem areas can be identified for use by multisystem programs, like MATE – Multi-Area Thévenin Equivalent [13], Multi-level MATE [14], and Latency exploitation [15]. These techniques allow the use of different time-step sizes in each separate subsystem, with further resynchronization [16] at specific times, thus optimizing the overall solution in EMTP-based off-line or real-time digital simulations [17].

The continuous-time eigenvalue calculation can also be used with (34) to find the exact values for optimum time step size and maximum simulation time. The eigenvalues calculated with this technique are exact.

Considering the branches associated with inductive and capacitive components and with transmission lines, for each complex continuous-time eigenvalue in (35), calculates the corresponding exact time constant as in (36), and damped oscillation period as in (37).

\[ \lambda_i = \sigma_i \pm j \omega_{di} \]  \hspace{1cm} (35)

\[ \tau_i = -\frac{1}{\sigma_i} \]  \hspace{1cm} (36)

\[ Tn_{di} = \frac{2\pi}{\omega_{di}} \]  \hspace{1cm} (37)
With these time values organized in ascending order in a vector \((\mathbf{T})\), we can now derive the exact optimum time step size and the maximum simulation time (14) and (15) as in the simplified procedure first presented.

By applying (29)-(34) to the case of Fig. 2 results:

\[
\begin{align*}
\lambda_1 &= -10 + j26.4575 \\
\lambda_2 &= -10 - j26.4575 \\
\tau_1 &= \tau_2 = \frac{1}{10} = 100\text{ms} \\
Tn\delta_1 &= Tn\delta_2 = \frac{2\pi}{26.2475} = 237.48\text{ms} \\
\Delta t &= \frac{1}{100} \times \min(\mathbf{T}) = \frac{1}{100} \times 100\text{ms} = 1\text{ms} \\
t_{\text{max}} &= 5 \times \max(\mathbf{T}) = 5 \times 237.48\text{ms} = 1.19\text{s}
\end{align*}
\]

In this simple case, when comparing the results obtained by (25) with (42) with the results obtained by (26) with (43), one has to assess the additional computational effort of performing the most accurate analysis with that of the simplified analysis of section II. More complex cases have to be developed to further assess the validity of the proposed algorithms.

The most important issue, however, is the emphasis on the learning process aiming to optimize the analysis of the transient simulation. If optimal time step size is not used, relevant phenomena may not be represented in the simulation.

Moreover, if optimal maximum simulation time is not used, overvoltages and or overcurrents which may arise along time due to travelling wave and or resonance phenomena may not be properly considered in the engineering insulation study or overcurrent protection coordination, eventually resulting in time, equipment or system failures with technical, economic, environmental or human losses.

V. CONCLUSIONS

This paper has provided algorithms to automatically determine the step size and the simulation length in EMTP-type of programs. A simple empiric method was proposed to calculate an estimate for the optimal time step size \((\Delta t)\) and for the maximum required simulation time \((t_{\text{max}})\). This would help the user by e.g. minimizing running time or providing a warning if an inappropriate time-step has been set.

A more exact algorithm, based on the determination of the circuit continuous time eigenvalues from the circuit parameters and the network topology is also suggested. This algorithm does not make the simplifications of the empirical method and has the potential of providing very accurate pre-determinations of the optimum time step and solution times.

User-independent determination of the simulation parameters is of particular importance for new users of EMTP programs. Existing programs require the user to specify these parameters, which can require extensive experience in performing transient simulations. A future challenge will be the extension of these techniques to adapt the time step size dynamically in EMTP programs as the topology and parameters of the system change during the simulation.

REFERENCES