Abstract – In this paper, a new methodology to assist system planners is proposed, which is able of providing a set of good solutions to the addition of new transmission lines, in order to reinforce the sub-transmission network. To circumvent the combinatorial explosion of reinforcement alternatives, a constructive heuristic algorithm is used for adding new branches to the sub-transmission grid. Based on performance indices to measure the attractiveness of new branches and using an expansion tree, the proposed algorithm is able to capture the combined effect of reinforcement additions. For selecting the best alternative, from the set of good reinforcement solutions, the planned network performance in meeting the future load demand is evaluated, considering the following operational aspects: ohmic losses, reliability, branch loading condition, and voltage profile. The evaluation of the performance indices and all operational aspects is achieved by applying a non-linear AC model-based power flow. For the reliability assessment, an enumeration technique is used to select system states. A real sub-transmission Brazilian system belonging to CEMIG utility is used to validate the methodology and the results are presented and extensively discussed.

Keywords – Constructive heuristic algorithm, reliability, sub-transmission expansion planning, transmission ohmic losses.

I. INTRODUCTION

Planning the expansion of sub-transmission networks consists of defining which reinforcements will be needed to meet future demand, maintaining the levels of quality and reliability of the electricity supply at minimum investment cost. The technical literature shows that few studies have been exclusively dedicated to the planning of sub-transmission systems [1]-[3]. Clearly, the techniques developed for the transmission expansion planning [4]-[6] can be employed in sub-transmission networks, since these systems have similar characteristics. However, one must be cautious when extrapolating the use of these techniques, since, in general, sub-transmission systems are composed by transmission lines (TLs) of shorter length with lower voltage level, have a less intense meshed network, and low number of generators connected to them.

Some recent studies have applied new techniques in sub-transmission systems [7]-[9] and may be cited as successful initiatives. These methods were based on linear DC network models to enable the handling of a huge search space and/or modeling of uncertainties. However, the expansion of a sub-transmission system, which involves a lesser meshed network than that presented by a standard transmission system, has peculiar features, not being able to ensure the “N-1” criterion; an assumption that has not been used in these recent studies. Thus, the consideration of reliability in the sub-transmission planning should be associated to the definition of minimum performance criteria, more flexible, but capable of ensuring the robustness of the planned configuration [10].

This paper proposes a new methodology to assist system planners, which is able of providing a set of good solutions to the addition of new TLs, in order to reinforce the sub-transmission network. For the addition of new reinforcements, the construction of new branches as well the duplication of existing circuits are both considered. To circumvent the combinatorial explosion of alternatives, a constructive heuristic algorithm is used for adding new circuits to the sub-transmission grid. Based on performance indices to measure the attractiveness of new branches and using an expansion tree, the proposed algorithm is able to capture the combined effect of reinforcement additions. The robustness of the system planned configurations is measured in terms of the following operational aspects: ohmic losses, reliability, bus voltage profile, and circuit loading levels. The evaluation of all these aspects is performed by applying a non-linear AC model-based power flow. For the reliability assessment, an enumeration technique is used to select system states. A real sub-transmission system of a Brazilian utility company, CEMIG, is used for testing and validating the proposed methodology. All obtained results are extensively discussed.

II. PROPOSED METHODOLOGY

A. General Aspects

The proposal planning methodology seeks to identify good alternatives for expansion in sub-transmission systems. In general, these systems use lower voltage levels as compared with the very high voltage level of the basic grid, and are composed of less dense meshed networks with several radial TLs. Due to these characteristics, sub-transmission systems display higher ohmic losses than transmission systems. It is also implicit that the reduction of ohmic losses can be primarily considered for the definition of an initial set of good alternatives to reinforce the system. It is relevant to mention that the minimization of active power losses produces more robust sub-transmission systems capable of absorbing the annual demand growth, besides enabling cost reductions with the generation of electricity. From the planning point of view, including both operation and expansion, it is also possible to highlight the following benefits:

This work was partially supported by the following Brazilian research institutions: CNPq, CAPES, and FAPEMIG.
• Obtaining more reliable systems – there will be a better balance in the distribution of power flow among the circuits, producing a larger reserve capacity, which will be useful for the network withstand contingencies;
• Reduction/optimization of future investments – as a result from greater robustness of the sub-transmission system, it is expected that the number/cost of the circuits to be added in future planning be lower;
• Postponement of investments – due to the larger reserve capacity, the sub-transmission grid can better accommodate the annual load growth, regardless of the addition of extra reinforcements in a given year.

The allocation of reactive support is not part of the scope of this paper. It is expected, however, that the selected alternative based on ohmic losses also provide benefits for other operational aspects of the system, as the circuit capacity loading level and bus voltage profiles, which make the system less dependent of ancillary services to support reactive power.

B. Performance Indices

For the identification of more promising new and/or existing branches, i.e., those that will produce more significant impacts on the operational aspects, two performance indices are used. They are calculated from the results of an AC power flow analysis and geo-referenced data of substations.

The first index \( \text{Ind}_\theta \), defined by (1), is directly related to the angular difference \( (\Delta \theta_{ik}) \) presented by the bus voltages \( (i \text{ and } k) \) connection of the branch.

\[
\text{Ind}_\theta = \Delta \theta_{ik} / \ln L_{ik}
\]  

(1)

Indirectly, this index considers the effects of the TL candidate reactance (i.e., that can be added to the branch), as well as the cost of investment, which will be proportional to the length branch \( (L_{ik} \text{ in km}) \), whatever the voltage level is. However, it is necessary to apply the logarithm to mitigate the impact of the length (or cost) of the branch on the index value. To take into account the pathway of a TL candidate, its length is artificially increased by 20% of the value obtained from the geo-referenced database. Obviously, the planner can apply additional adjustments to the obtained lengths, especially for TL candidates that achieved good rating by the proposed indices. In this work, only branches connecting buses with 138 kV are considered. Variations of this index, which explicitly include the reactance and the cost of investment, can be used to consider interconnection through branches with different voltage levels (e.g., 69 kV buses).

Due to the higher \( P - \theta \) coupling presented in the power flow problem [11], the \( \text{Ind}_\theta \) index will point out candidates TLs with high potential for the passage of active power flows. Thus, it is expected that such TLs establish the connections which are able to redistribute the active power in the system, especially in its vicinity, with impacts that may be significant for all operating aspects considered in this work.

The second index \( \text{Ind}_V \), described by (2), relates the modular difference of the voltage magnitudes of the branch connections \( (\Delta V_{ik} \text{ given in percent}) \) with the logarithm of its length (provided in km), i.e.,

\[
\text{Ind}_V = \frac{\Delta V_{ik}}{\ln L_{ik}}
\]  

(2)

The use of the logarithm of the length follows the same assumptions of the first index \( \text{Ind}_\theta \). The use of modular voltage differences, however, is inspired by the fact that the \( Q - V \) coupling is more intense than the \( Q - \theta \) coupling in power flow problems [11]. The candidate TLs with higher \( \text{Ind}_V \) index values will have constructive impacts on redistributing the reactive flows, enhancing bus voltage profiles and circuit loading levels, thus reducing the amount of ohmic losses. It is also compelling to expect that such TLs have positive impacts on voltage problems under contingencies, further contributing to the system reliability.

C. Proposed Algorithm

This subsection describes the Constructive Heuristic Algorithm (CHA), based on a tree search and proposed for obtaining the best expansion TL alternatives. Most network operating aspects to be considered in this work are evaluated by the handling of results obtained through power flow runs. For this purpose, a commercial AC power flow program provided by CEMIG concessionary of energy is used. It is noteworthy that the central idea of this work is to propose a methodology based on tools routinely used by electric utilities, increasing the chances of its assimilation by system planners.

The proposed algorithm is able to capture the attractiveness of the branches for the initial base system network configuration and for the later conditions, i.e., after adding reinforcements to the grid. The idea is to select candidate branches for both the base configuration and all possible reinforced configurations, i.e., after the first reinforcement, second one, and so on.

In the initial phase of the algorithm, actions are taken to eliminate the presence of similar reinforced configurations, a highly detrimental condition to the performance of the methodology, since it reduces the amplitude of the tree search. Therefore, it is necessary to identify the presence of similar new branches (i.e., having the same terminal bus, where the interconnected buses are nearby terminals, usually located in the same municipality). Furthermore, it must choose among identified similar branches the one that will be used in the algorithm. The criterion used in this work is to select the branch with the largest sum of normalized (i.e., in per unit of the highest value obtained for each index) \( \text{Ind}_\theta \) and \( \text{Ind}_V \) indices. It has to be mentioned that, at the end of the search process, for each of the best obtained configurations, the recovery of similar branches, which have been discarded, is performed. Thus, every configuration similar to any of the best solutions, found by the algorithm at that stage, will be evaluated and, in case it presents a better performance, it will replace the previous one.

In order to clarify the search tree process, the CHA steps that are used to build the expansion tree model are presented as follows:

i) Determine the set of candidate branches receiving reinforcements from the specifications provided by the system planner. The branches that will be part of this group must have length within the established range (e.g., 8 to 150 km). In addition, the existing branches must present power flows, in per unit (pu) of its own capacity, greater or equal to the limit set by the planner (e.g., 0.6 pu);
ii) Eliminate the presence of similar new branches within the set of candidate branches. To do this, one must assess the base configuration of the sub-transmission system and calculate both indices $Ind_0$ and $Ind_1$ of each new branch. Then, using as the criterion the largest normalized sum of these indices, it should be selected for each pair or group of similar branches the one to be kept.

iii) Create the first level of the expansion tree (i.e., with only one reinforcement), selecting the $NB$ (quantity defined by the planner, e.g., 6 branches) more attractive candidate branches, in terms of performance indices. Half of the selected branches should be pointed out by the highest values of the $Ind_0$ index, the other half similarly established by the $Ind_1$ index. Set the current level of the expansion tree as being the first one;

iv) Evaluate each configuration of the current level of the tree, in order to calculate the values of $Ind_0$ and $Ind_1$ indices, as well as the amount of ohmic losses;

v) Select the configurations that show the lowest amounts of ohmic losses, to compose the set of best candidate configurations at the current level of the expansion tree;

vi) From each configuration assessed in step (iv), select by means of $Ind_0$ and $Ind_1$ indices a set with the same number of candidate branches ($NB$) used in step (iii), thereby creating configurations that may belong the next level of the expansion tree;

vii) Perform the natural pruning of the tree, eliminating repetitions (configurations with the same combination of reinforcements) among the configurations generated in step (vi). If after the natural pruning there is a number of configurations greater than the maximum value ($NC$) determined by the user (e.g., 300), evaluate all these configurations and make a forced pruning of the expansion tree, selecting only the $NC$ configurations with lower values of ohmic losses to be part of the next level of the tree;

viii) Increase the current level of the tree. If the final level of the expansion tree (i.e., one having the maximum number of reinforcements allowed by the planner) is reached, go to step (ix), otherwise, return to step (iv);

ix) Evaluate each configuration of the final level of the tree in terms of ohmic losses, selecting those presenting the smallest amount of network losses to compose all the best candidate configurations of the final tree;

x) Evaluate each candidate configuration regarding other operational aspects (i.e., reliability, bus voltage profile, and circuit loading levels);

xi) For the best evaluated candidate configurations in the previous step, generate and evaluate possible similar configurations.

It is important to note that steps (x) and (xi) of the proposed CHA can be applied to the obtained candidate configurations for any level of the expansion tree.

In the expansion tree of Fig. 1, the node at the top represents the base configuration (i.e., no reinforcements) of the system. Just below, one can see the configurations with only one reinforcement, obtained from the addition of a TL to each branch selected at step (iii) of the algorithm. In the next level below, there are configurations with two reinforcements, but still, without the application of step (vii) of the algorithm (tree pruning). The C1.1 to C1.6 configurations are generated from C1 configuration. This notation applies to other settings. Thus, for example, C6.1 represents the first configuration selected from C6. Based on Fig. 1, one can easily extrapolate new branches (for three or more additions) of the expansion tree.

Finally, it is noteworthy that the CHA proposed in this paper represents a parallelization (with tree branches) of the sequential algorithm for adding reinforcements, proposed in [3], besides allowing the addition of double circuits to new branches and a second TL in existing branches (i.e., containing a single circuit).

### III. APPLICATION

#### A. System Characteristics

The system used to represent the sub-transmission grid of CEMIG incorporates all the essential part of the bulk Brazilian network. The whole system has 4998 buses, with 742 generating buses (including small hydro and independent producers), and 7061 circuits, from which 4387 are transmission lines and 2674 are transformers. The total load of this system reaches the amount of 71,610 MW. Only 138 kV circuits are considered for reinforcing the system. The costs of TLs, which vary accordingly to their length, are presented in Table I, and the cost of the bay is US$ 491,000.00. Although only one type of cable has been used in this study, the proposed CHA-based methodology allows that different cables with different capacities and investment costs be considered. Finally, all results presented in the next sections were obtained using an i7 processor with 2.93 GHz.

#### B. Methodology Validation

To validate the proposed methodology (CHA Case), the best solutions obtained at each level of the expansion tree must be compared with the results from an exhaustive search of tree named as Reference Case (Ref. Case), which considers all possible combinations of reinforcements using the candidate branch set defined in steps (i) and (ii) of the algorithm, having the amount of ohmic losses as a selection parameter. This subsection considers a small size problem, for which only the sub-transmission grid named Eastern Area will receive reinforcements. Additionally, in order to prove the efficiency of the proposed performance indices (bearing in mind the

<table>
<thead>
<tr>
<th>Range</th>
<th>Single (US$/km)</th>
<th>Double (US$/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 10 km</td>
<td>99.840.74</td>
<td>132,409.28</td>
</tr>
<tr>
<td>≥ 10 and &lt; 20 km</td>
<td>81,696.59</td>
<td>114,401.05</td>
</tr>
<tr>
<td>≥ 20 km</td>
<td>73,658.89</td>
<td>99,503.17</td>
</tr>
</tbody>
</table>
identification of reinforcements that have the greatest impact on the amount of losses, a third case (*CHA*<sub>lost</sub> Case) is considered, where the constructive heuristic algorithm is adapt for enabling only the use of ohmic losses to select the best reinforced configurations.

The Eastern Area, used in this subsection, consists of 285 buses (21 buses of 138 kV with geo-referenced coordinates), among which 11 are generation buses, with a total installed capacity of 630 MW. Moreover, the network of the Eastern Area has 311 lines, from which only 22 are transmission lines linking 138 kV geo-referenced buses. The set of candidate branches, obtained by steps (i) and (ii) of the algorithm, contains 5 existing branches and 34 new ones. For the calculation of reliability indices, re-dispatch of generation and transmission equipment failure are allowed only in Eastern Area (region of interest), whose total load for the year horizon is 1300 MW. From 311 circuits belonging to the Eastern Area, only 110 are considered in the operating (up) and repair (down) process. This occurs as a function of radial circuits with smaller or equal to 34.5 kV voltage levels, which are considered as being by assumption as totally reliable. For clarity, Figure 2 shows a simplified line diagram, containing 138 kV geo-referenced buses of the Eastern Area and their connections. This diagram also shows the reinforcements found in the best solution.

To implement the *CHA* and *CHA*<sub>lost</sub> cases, a number of best attractive candidate branches (NB) equal to 6 is used. As for the exhaustive search (*Ref. Case*), no restriction for the NB parameter is imposed on the tree construction process; only the natural pruning of step (vii) is applied.

Table II presents the results obtained in the three cases, considering the first level of the expansion tree. Clearly, the six selected configurations (reinforcements) for *Ref.* and *CHA*<sub>lost</sub> Cases are the same because they correspond to configurations with lower amounts of active losses, among the 39 possible configurations reinforced for the first level of the tree. In relation to the *CHA* Case, one realizes that the performance indices are able to point out three among six reinforced configurations with smaller amounts of active losses. It is noted that (in this and other tables) when two or more configurations provide the same amount of active losses, they are decreasingly ordered in terms of their investment costs.

To prove the efficiency of *Ind<sub>a</sub>* and *Ind<sub>v</sub>* indices, Table II includes the respective values obtained for the corresponding reinforcements of each branch. Note that, the branch 435-540 is very attractive, being selected in the first place by the two indices. Due to this coincidence, the other 3 branches (523-435, 506-532, and 620-660) were selected by *CHA* Case, reflecting the applicability of these indices in identifying branches with high potential to reduce ohmic losses.

On the second level of the expansion tree, there is a total of 775 possible configurations (i.e., combinations of 2 reinforcements), all evaluated by the *Ref. Case*. Among 36 possible configurations generated by the proposed *CHA* (i.e., 6 best reinforcement options for each of the 6 configurations selected at the first level of the expansion tree), only 32 are kept after the natural pruning procedure, while using the *CHA* Case. As for the *CHA*<sub>lost</sub> Case, 23 configurations remain after the pruning process. However, it was necessary to evaluate a total of 218 configurations to obtain the corresponding amounts of active losses, necessary for the identification of NB reinforcements leaving each node (configuration) of the expansion tree. Table III summarizes the results obtained with the three ana-
lyzed cases. Only the 10 best solutions, in terms of active power loss (Ref. Case) are presented. Among these, four solutions are found by the CHA Case and 6 by the CHA\textsubscript{loss} Case. The average values of active power losses given in Table III are obtained for the 10 best solutions of each case.

Table IV presents 10 configurations involving 3 reinforcements (third level of the expansion tree) that provide the smallest amounts of active losses. As adopted in Table III, the average values of losses are obtained for the 10 best solutions for each case. Altogether, given the set of candidate branches defined by steps (i) and (ii) of the algorithm, there are 10431 enhanced configurations formed by three reinforcements, all evaluated by Ref. Case. As for the CHA and CHA\textsubscript{loss} cases, 145 and 812 configurations were evaluated, respectively. The proposed methodology (CHA Case) found 6 out of 10 configurations with the lowest power losses. Considering losses as the selection criterion, i.e., CHA\textsubscript{loss} Case, it yields 10 successful hits. It is worth noting the improvement in quality of the solutions offered by the proposed CHA-based method, whose average loss solutions became very close to that obtained by the Ref. Case (all combinations).

For the 4\textsuperscript{th} level of the tree, the combination of reinforcements from the set of candidate branches results in 106 714 different enhanced configurations. When employing the proposed methodology, i.e., CHA Case, 651 configurations are obtained after applying the proposed indices (Ind\textsubscript{d} and Ind\textsubscript{y}), and the subsequent natural pruning of the expansion tree. These 651 configurations must be evaluated to apply a forced pruning, selecting, by step (vii) algorithm, 300 (NC) configurations with smaller amounts of active losses. As for the CHA\textsubscript{loss} Case, 2737 configurations must be evaluated for the application of step (vi) of the algorithm (i.e., to use the losses to replace the performance indices as the selection criterion). As for the CHA Case, at the end of step (vii), 300 configurations will form the fourth level of the expansion tree.

Among all the configurations obtained from 4 reinforcements, those ten which provide lower amounts of active losses (Ref. Case) are shown in Table V. It is found that the proposed methodology (CHA Case) gave an excellent result, finding 6 among the 10 configurations with the lowest losses, in addition to providing an average value of losses (for the 10 best solutions found by CHA Case) only 0.2 MW above to the average obtained by the Ref. Case. The eight best solutions found by the CHA\textsubscript{loss} Case, six are also obtained by the CHA Case. Note that the top three solutions are correctly found by the proposal CHA methodology.

To obtain the results up to the 4\textsuperscript{th} level of the expansion tree, the CPU processing times required for the CHA, Ref., and CHA\textsubscript{loss} cases are, respectively, 25, 2006, and 83 minutes. The speed-ups achieved by the proposed methodology in relation to the Ref. and CHA\textsubscript{loss} cases are 80.4 and 3.3, respectively. In order to illustrate the step (x) of the proposed CHA, for the analysis of the candidate configurations, 3 configurations with 4 reinforcements (4\textsuperscript{th} level of the expansion tree) are used, since they have presented the smallest amount of active losses within the adopted investment limit of 37 million dollars. Only the configurations found by the proposed methodology (CHA Case) were used. Table VI shows the amounts of active losses and investment costs of the best candidate configurations. C1 and C2 correspond, respectively, to the 1\textsuperscript{st} and 6\textsuperscript{th} configurations of the CHA Case (1\textsuperscript{st} and 9\textsuperscript{th} configurations of Table V). The C3 configuration is the 7\textsuperscript{th} with the lowest active losses, found by the proposed method (12\textsuperscript{th} found by the Ref. Case), which is formed by the addition of a double circuit between buses 435 and 540, and two single circuits, which interconnect buses 660-680 and 607-850. It is noteworthy that among all the configurations in Table V, only the 1\textsuperscript{st}, 7\textsuperscript{th}, and 9\textsuperscript{th} did not violate the investment limit. Therefore, only one of these configurations (the 7\textsuperscript{th}) was not selected by the proposed methodology (CHA Case). Clearly, the 7\textsuperscript{th} configuration of Table V presents smaller amount of active losses (16.4 MW) compared to the value provided by C3 configuration. However, the investment cost (34522 million) is significantly higher.

In assessing the reliability of the best candidate configurations, a state enumeration process considering only the first-order TL contingencies of the Eastern Area 138 kV is used. For the base configuration, the assessed state space reached the probability of 0.9984 (i.e., almost the probability space). In other configurations analyzed this probability reached 0.9980. The results obtained for the base, C1, C2, and C3 configurations, expressed in terms of the LOLE (loss of load expecta-

Table IV

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>Losses (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From/To</td>
<td>CHA Case</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------</td>
</tr>
<tr>
<td>435/540</td>
<td>16.4</td>
</tr>
<tr>
<td>Average</td>
<td>16.4</td>
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</tbody>
</table>

Table V

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>Losses (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From/To</td>
<td>CHA Case</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------</td>
</tr>
<tr>
<td>435/540</td>
<td>15.8</td>
</tr>
<tr>
<td>Average</td>
<td>15.8</td>
</tr>
</tbody>
</table>
load frequency), and LOLD (loss of load duration) indices [12], are shown in Table VII. In general, the reinforced configurations exhibit a significant improvement in the reliability indices, especially for EENS. Undoubtedly, C2 configuration shows the best performance considering the reliability aspect, with reductions of: 14% (LOLE), 24% (EENS), and 17% (LOLF), in relation to the base case (indices). These results clearly show a direct relationship between the reduction of active power losses of the system and reliability improving.

Figure 3 shows the probability density functions of bus voltage magnitudes and circuit loadings of the 138kV circuits. One can perceive an improvement of the voltage profile and the relief of circuit loading for the enhanced configurations. The increasing in the voltage profile is indicative of fewer requirements for reactive support for C1 to C3 configurations. Observing a smaller number of circuits with loadings greater than 0.5 pu, it suggests that the reinforced configurations represent, indeed, less stressed networks. Bearing in mind this latter aspect, C1 configuration presents slightly higher performance than the others. The probability densities in Fig. 3 confirm the assertion that configurations with lower active losses tend to outperform in the operational aspects of bus voltage magnitude and circuit loading profiles. Considering all operational aspects and investment, C2 configuration (whose reinforcements are shown in Fig. 2) should be elected as the best.

To accomplish step (xi) of the algorithm, all similar candidate configurations of Table VI were assessed. Altogether 13 configurations were evaluated, with 7 of them similar to C1 configuration. The C2 and C3 Configurations had each one 3 similar settings. It is important to mention that no similar configuration provided a smaller amount of active power losses than the one obtained for the respective candidate configuration. This fact is another indicator of the effectiveness of the proposed indices in selecting the most impactful branches in terms of reduction of active power losses.

### C. Large Size Power Networks

Aiming to illustrate the use of the proposed methodology in large power grids, this subsection describes the investigation of reinforcements not only in the Eastern area of CEMIG system, but also including two neighboring areas: Central and Northern. Therefore, the number of buses and new branches in the 138 kV network are now much larger. In this new problem, the area of interest, called ECN Area, has 81 buses of 138 kV (with geo-referenced coordinates), with 21 in the Eastern Area, 41 in the Central Area, and 19 in the Northern Area. Altogether, the ECN Area has 365 buses among which 13 buses are generation, for a total installed capacity of 1 760 MW. Furthermore, the respective network has 451 circuits, where 88 are transmission lines connecting 138 kV buses with coordinates. The total load for the horizon year is 3 813 MW. An illustrative net map of the ECN Area is shown in Fig. 4.

In the ECN Area, the number of new candidate branches (N_{NB}) is 458, while the number of existing branches (N_{EB}) is equal to 11. For this larger problem, it is no longer possible to perform an exhaustive search of tree (Ref. Case). Additions up to 6 reinforcements are allowed and, thus, 15 424 108 676 322 configurations (see Equation 3, which calculates the total number of configurations or nodes of the expansion tree) would have to be evaluated, which would require about 587 000 years of simulation using the same 7 – 2.93 GHz processor. Therefore, only the CHA and CHA_{bas} cases are considered. The parameters NB and NC are the same used in Section III.B (6 and 300, respectively). For these cases, the processing CPU times needed to evaluate up to the sixth level of the expansion tree reached 164 and 8073 minutes, respectively. The speed-up presented by the proposed methodology in relation to the CHA_{bas} Case is 49.1.

The total number of combinations (TNC) of reinforcements (configurations or nodes of the expansion tree with an exhaustive search), involving the addition of TLs up to the maximum number of reinforcements (M_{NREF}), can be calculated by combinatorial analysis using the following expression:

$$TNC = \sum_{N_{REF}}^{M_{NREF}} \left( \sum_{i=0}^{N_{EB}} \left( \begin{array}{c} N_{NB} \\ i \end{array} \right) \times \left( \begin{array}{c} (N_{NB} - i) + N_{EB} \\ N_{REF} - 2 \times i \end{array} \right) \right)$$

(3)

where: N_{REF} represents the number of reinforcements (TLs) added to each configuration, and M_{N} is the maximum number of double reinforcements that can be added and is obtained by the integer resulting from the quotient between N_{REF} and 2.

In relation to the best configurations obtained at each level of the expansion tree, there is lesser coincidence among the results obtained in the CHA and CHA_{bas} cases. However, as highlighted in Table VIII, there is a close proximity between the average active power losses presented by the best solutions of both cases (6 for Level 1 and 10 for the others). The same is

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Losses (MW)</th>
<th>Investments (10^6 US$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>15.8</td>
<td>36.731</td>
</tr>
<tr>
<td>C2</td>
<td>16.6</td>
<td>32.057</td>
</tr>
<tr>
<td>C3</td>
<td>16.8</td>
<td>31.971</td>
</tr>
</tbody>
</table>

**Table VI**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>LOLE (h/y)</th>
<th>EENS (GWh/y)</th>
<th>LOLF (occ./y)</th>
<th>LOLD (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>231.86</td>
<td>4.82</td>
<td>64.79</td>
<td>3.58</td>
</tr>
<tr>
<td>C1</td>
<td>213.60</td>
<td>3.97</td>
<td>58.72</td>
<td>3.64</td>
</tr>
<tr>
<td>C2</td>
<td>199.07</td>
<td>3.68</td>
<td>53.79</td>
<td>3.70</td>
</tr>
<tr>
<td>C3</td>
<td>211.63</td>
<td>3.73</td>
<td>59.51</td>
<td>3.56</td>
</tr>
</tbody>
</table>

**Table VII**

![Fig. 3. Probability density functions.](image-url)
not true with the average investment. Clearly, the proposed methodology provides solutions with substantially lower level of investment. For the 6th level of the expansion tree, one has a reduction of US$ 10.81x10^6, which corresponds to 22.5% of the average investment of CHACase.

Unfortunately, there are no approaches in the literature applied to sub-transmission systems that enable performance comparisons with the proposed methodology. However, it is noteworthy that the computational gain provided by the use of Ind_a and Ind_v indices, in this second case involving a large network, is very significant (49.1 times faster). It should also be noted that the application of metaheuristics techniques to search for the best solutions to this problem is currently under analysis.

IV. CONCLUSIONS

In this paper, a new methodology for planning sub-transmission systems is proposed. It uses a constructive heuristic algorithm based on an expansion tree. For signaling the best reinforcements in the network, two new performance indices are used. It is worth mentioning that the planning strategy adopted is based on the main transmission network designed for the horizon year, which is more suitable for networks in countries with large territory, as the Brazilian case. Another relevant aspect is the AC network modeling, which is essential for analyzing both active and reactive power flows and assessing all performance characteristics necessary in the medium-term planning, i.e., between 5 and 10 years ahead.

The obtained results confirm the efficiency of the proposed indices to identify major reinforcements, in order to reduce active power losses, while minimizing investment costs. It was possible to ensure that configurations with reduced amounts of losses have good performances, in terms of other operating aspects such as reliability, voltage profile and circuit loadings. Therefore, the planned configurations are proved to be more robust and capable of absorbing the annual demand growth. High computational efficiencies were obtained, which makes the methodology appropriate to deal with large real networks.

Table VIII

<table>
<thead>
<tr>
<th>Level</th>
<th>Average Losses (MW)</th>
<th>Average Investments (10^5 US$)</th>
</tr>
</thead>
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<tr>
<td></td>
<td>CHA Case</td>
<td>CHAAS Case</td>
</tr>
<tr>
<td>1</td>
<td>71.7</td>
<td>66.8</td>
</tr>
<tr>
<td>2</td>
<td>65.8</td>
<td>63.2</td>
</tr>
<tr>
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<td>62.0</td>
<td>60.3</td>
</tr>
<tr>
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<td>56.7</td>
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<tr>
<td>6</td>
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<td>55.3</td>
</tr>
</tbody>
</table>

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REFERENCES